Hybrid OLS for uncertainties estimation in direct shear testing

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Abstract

Measurement uncertainty (MU) estimation is not prescribed by current direct shear tests (DST) standards, even though MU estimation is a powerful tool for risk mitigation in geotechnics. The hybrid Ordinary Least-Squares (OLS) approach is advanced to overcome this shortcoming, denoted as HOLS. OLS, Iterative Weighted Least Square (IWLS) and Weighted Line of Organic Correlation (WLOC) approaches are proven not suitable for MU estimation in DST practice, as OLS largely overestimates MUs, while IWLS and WLOC strongly underestimate MUs. HOLS provided more reliable MUs than OLS, IWLS and WLOC. HOLS was implemented with a matrix-based algorithm and it was validated by inter-comparison between published data and its outputs. *The worst case* strategy is proposed in case of feeble knowledge about errors correlations. Emphasis is given to situations where uncertainty must be used to evaluate the soil reinforcement rate with polyester fibers. HOLS is suitable to many similar metrological cases.

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Keywords: direct shear test; measurement uncertainty; hybrid ordinary least-squares approach; error-in-both-variables; error correlation, heteroscedasticity

1. Introduction

Shear strength is responsible for the soil ability to withstand applied loads and it is regarded as the most important engineering property of soil [1, 2, 3]. The knowledge of the shear strength parameters is essential for the analysis of the subsoil bearing capacity, slope stability and earth pressure [1, 2, 3, 4]. The shear strength parameters of a certain type of soil in an area varies as the soil structure becomes more complex and its deformation properties and shear strengths depend on many factors (mineral composition, water content, particle size, loading speed etc. [4, 5, 6, 7]. Direct shear tests (DST) and triaxial tests are the most appropriate for determination of the soil shear strength parameter [5, 6, 7, 8, 9, 10, 11]. Nevertheless, the DST is mainly performed as its performance/cost ratio is superior to the triaxial one [4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

The laboratory DST practice ignores the measurement uncertainty (MU) that could be assigned to the outcomes [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], even though the standards EN ISO/CEI 17025 [17] and ISO/CEI Guide 98-3 [18] standards specify that every numerical output of a measurement process is affected by an uncertainty in a certain range. In the view of these standards, the quality of a numeric measurement result gets higher as its assigned MU value gets smaller. The referential standards [17, 18] emphasize that the measurement results cannot be compared to specified values or among them without taking into account their MUs. Furthermore, there are many soil improvement techniques in which various materials are added to soil to increase the soil shear strength parameters [8, 9, 19, 20, 21]. In these cases, it is necessary to find the optimal amount of the reinforcing materials, so a comparison of the improvement rates is carried out. In many cases, the improvement rate can be smaller than the MU value; however, without a proper MU estimation it is quite groundless to assess the optimal amount of the materials. In this regard, the authors in [19] used DST to evaluate the improvement of the angle of internal friction (ϕ) of sand reinforced with Polypropylene fibers. The values of ϕ introduced by the authors are 42.3° (without fibers), 42.1° (0.1% fibers), 41.8° (0.25% fibers), 40.6° (0.5% fibers) and 40.4° (1% fibers) which probably will be smaller than their assigned MUs. Authors in [20] introduce the improvement of kaolin slurry treated with cement, where in Fig. 8 of the paper one can see that improvement of ϕ and cohesion (c) between mixture with 5% and 7.5% cement content is about 1° and $3kPa$, respectively. Therefore, determination of MU can be useful since such small changes can be smaller than their assigned MUs. Authors in [21] introduce comparative measurement of compaction impact of clay stabilized with cement, peat ash and silica sand where parameters of shear strength were obtained by DST in accordance to the standard of ASTM D3080. In Tab. 3 of the paper one can see that improvement rate of the angle of internal friction and cohesion is very small, often about $1° - 2°$ and $1kPa$–$2kPa$. Again, the estimation of MU can be useful since such small changes can be smaller than MU.

The actual DST practice across the EU is not fully harmonized. Thus, in Romania, the DST standard method is specified in the standard STAS 8942/2-82 [14], which came into force about 40 years ago. This standard does not address the problem of MU estimation or of the validity of the measurement outcomes.

The Slovakian standard STN 73 1030:1988 [22], prescribes 4 regression points for a better estimation of the calibration line slope (β), of the ϕ and of the regression line intercept (c) values, based on the OLS approach. In the case of negative c value, the regression line is forced to pass through the (0, 0) point. This standard does not claim to estimate MU in DST, but it imposes a lower limit of the regression coefficient $r_a = 0.95$ for a 5% significance level and $r_a = 0.9588$, for 1% significance level. In the case of $r \leq r_a$ it is necessary to verify whether there is any reason to exclude some specimens from evaluation. If after verification the condition $r \geq r_a$ is still not fulfilled, it is necessary to find another fit line, which would better fit the effective shear strength.

The Polish standard PN-88/B-04481 [23] prescribes 5 regression points for the estimation of the shear strength parameters. Also, this standard provides formulae for experimental standard deviations of β, c and, subsequently, ϕ. According to this standard, one should check whether the deviation of the experimental point differs more than 25% from the calculated one. If yes, it is necessary to exclude that particular observation from evaluation and to use the next one to ensure that the conditions are fulfilled. The PN-88/B-04481 standard can be considered the precursor of the modern trend in DST practice, since it addresses the standard deviation of the DST results and draws attention to the reliability of DST results for the geotechnical design.

The ASTM D3080 [15] avoids addressing the uncertainty problem of the DST results. It specifies that *test data on precision are not presented due to the nature of the soil or rock, or both materials tested by this standard [...], it is either not feasible or too costly to produce multiple specimens that have uniform physical properties. Any variation observed in the data is just as likely to be due to specimen variation as to operator or laboratory testing variation.*

Even the latest standard ISO 17892-10:2018 [16] does not address in an explicit manner the MU estimation in DST.

Though the DST standards do not require MU estimation, the problem of MU estimation in DST practice was considered by many geotechnical specialists, being aware of the great importance of MU for decision making in their professional fields [8, 9, 11, 24, 25]. In this regard, the proper MU estimation for each soil type has to be a permanent concern for each DST laboratory, as neglecting MU or underestimating it can increase the risk of an improper decision regarding subsoil bearing capacity, slope stability, earth pressure, soil reinforcing applications etc.

All the DST standards addressed above [14, 15, 16, 22, 23] have in common the classical OLS for estimating the parameter of the calibration line ($\beta,\ \phi,\ c$). OLS makes use of a set of points in a Cartesian system i.e. $(x_i,\ y_i)$, $i = 1...n$, that are supposed to be in a linear relation $y_i = a + bx_i$. OLS is applicable when all y_i values are equally uncertain ($u(y_1) = u(y_2) = ... = u(y_n)$) and the uncertainties of x_i values are negligible [26]. When uncertainties of y_i values are not equal, but uncertainties of x_i values are negligible the literature recommends the Weighted Least-Squares (WLS) method [24, 25, 26, 27]. If both sets of x_i and y_i values have non-negligible uncertainties, then OLS is recommended to be replaced by other linear regression method [26, 27, 28, 29, 30, 31, 32, 33].

ISO/TS 28037:2010 [34] was issued to provide valuable solutions when one must perform linear calibration with uncertainties in both coordinates. This standard relies on the Iterative Weighted Least Square (IWLS) method based mainly on the works of D. York [27, 28, 33]. The IWLS method minimizes the sum of weighted distances between the line and the observed points (X_i, Y_i), $i = 1...n$. The work of York et al. [33] provides the relations for the DST parameters in case of correlated errors of X_i and Y_i values. In 2017, C. Delmotte [26] proved the better adequacy of the Weighted Line of Organic Correlation (WLOC) for airtightness measurements. One advantage of WLOC is that it minimizes the errors in both X and Y directions. Another advantage of WLOC is that it provides analytical solutions for the slope and the intercept.

The actual state of the art procedure in DST practice can be summarized as follows:

(i) DST standards [14, 15, 16, 22, 23] prescribe OSL for β and c estimation;

(ii) The MU estimation is not issued by these standards, but the geotechnical applications and researches need MUs values assigned to DST outcomes;

(iii) The uncertainty budgets assigned to DST measurands are larger and they imply treating errors in both linear regression variables.

The application of the likelihood method to heteroscedastic correlated errors in both regression variables is the natural solution to solve the MU estimation of DST outcomes, but this approach is complicated and does not yield analytical solution for the β and c measurands. Consequently, the evaluation of MUs of β and c implies an iterative computation procedure, hence it is unaffordable to many DST laboratory operators.

The EUROLAB Technical Report 1/2006 [35] suggests that the uncertainties related to parameters β and c can be estimated using uncertainties of both variables for the case of OLS calibration. Based on EUROLAB Technical Report 1, Nguyen et al. [7, 8, 9], have estimated the uncertainties of the calibration parameters using a correlated uncertainty approach. The correlation coefficients were considered constant for all coordinate pairs and equal to the correlation coefficient value assigned to the calibration line. The appliance of OLS to the case of linear regression with error in both coordinates apparently contradicts the consecrated scientific approach based on IWLS or WLOC [26, 27, 28, 29, 30, 31, 32, 33, 34].

On the other hand, the DST has at least 3 peculiarities:

(i) destructive test;

(ii) large uncertainty induced by the specimens itself (heterogeneity, instability, degradation, or ageing); and

(iii) few regression points (3...5) [5, 11, 12, 13, 14, 16, 22, 23, 36, 37, 38].

The destructive character and intrinsic heterogeneity of the specimen make it quite impossible to achieve an accurate MU estimation in DST practice. The smaller regression point number greatly increases the sensitivity of the calibration line parameters to the small inaccuracy in assigning the weights to the calibration points [8, 9]. In such a case, the hybrid strategies for the evaluation of MU mentioned in EUROLAB Technical Reports [35, 39] and the works of Nguyen et al. [6, 7, 8, 9] have led us to consider OLS as a provider of DST measurands values. Subsequently, to estimate uncertainties assigned to measurands based on the uncertainty propagation law given in [35]. This approach gives rise to a hybrid OLS, denoted HOLS, which makes the best compromise between the characteristics of the DST and of the necessity for a parsimonious estimation of the uncertainties of the slope and intercept. The HOLS is an improved version of the approach given by Nguyen [7, 8, 9], which is denoted GNA hereafter. In this regard, a comparative study is addressed in this paper using 6 methods to estimate the calibration line parameters in DST (β, ϕ, c, $u(\beta)$, $u(c)$) i.e. OLS, GNA, ODR (orthogonal distance regression), WLOC, IWLS and HOLS.

The main novelty addressed in the paper consists in demonstrating that, for DST practice, the HOLS is the best compromise between the outcomes and the costs. The HOLS takes into account all the contributors to the uncertainty budget of $\beta(\phi)$ and c. The HOLS approach can cope with heteroscedasticity in both variables. In order to apply the HOLS, a new and user-friendly algorithm was developed in Excel. This algorithm is based on the matrix data organization of the uncertainties, of the sensitivity coefficients and of the correlation coefficients of the input quantities as is suggested in [35, 36, 37, 38, 40].

The HOLS approach solves the issue of the proper assessment of the uncertainties of the DST measurands, provided that the covariance values of the input quantities are available. In the case of insufficient information about the correlations among input data, the paper recommends *the worst-case strategy* [35]. The paper argues for negative error correlations in DST measurements, rather than positive ones.

Emphasis is given to situations where MU has to be used to evaluate the improvement rate of soil reinforced with various materials (e. g. with polymeric fibers or chips, ashes, cement etc.) where the increase of soil shear strength parameters can be small and it is necessary to prove that the increase is really larger than the uncertainty.

2. Materials and methods

2.1. Materials

The paper aims to implement a new HOLS approach for a better assessment of the MU in DST. In this direction, the new approach was applied using the previously published results [7, 8, 9] so as to have proper comparative data. In this regard, the HOLS approach was applied to some DST results obtained in the Geotechnical Laboratory, Department of Geotechnics, Faculty of Civil Engineering, University of Zilina, Slovakia. A fully automatic large shear box apparatus – SHEARMATIC 300 (Wykeham Farrance, CONTROLS Group, Milan, Italy) was used to perform the DSTs. The results obtained on some reinforced soils with polyester fibers were considered as the MU plays a decisive role in assessing the research progress in this field.

2.2. Methods

To date, many linear regression methods have been developed to cope with different point spreading and with heteroscedastic uncertainties of both coordinates [7, 8, 9, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 39]. These methods use a set of n experimentally determined points in rectangular coordinates, generically denoted (x_i, y_i), $i = 1...n$ and search for the best straight line passing through them in term of slope (b) and intercept (a) i.e. $y = a + bx$. The methods can be classified based on their principle as: least-squares structural model, least-squares functional model, maximum likelihood, grouping, cumulant, moments or equation error [31, 32].

In this paper we address the representative least-squares structural models (OLS, IWLS, ODR) and the alternative WLOC [26, 33, 34, 37, 41]. The outcomes of any linear regression method are the line parameters a and b and their standard uncertainties, denoted $s(a)$ and $s(b)$ in most papers. To comply with DST practice and with VIM recommendations, we adopted the following notations: σ for x, τ for y, c for a, β for b, $u(c)$ for $s(a)$ and $u(\beta)$ for $s(b)$ [14, 22, 35, 42, 43].

The principles of the methods we studied and their main interesting features for the paper goals are presented below.

2.2.1. OLS method

OLS is the standard method applied in DST practice designed to give the best straight line [5, 6, 7, 8, 9, 14, 15, 16, 22, 23, 44]:

$$\tau = \sigma \cdot \beta + c = \sigma \cdot \tan(\phi) + c \tag{1}$$

where β is the slope; ϕ is the angle of internal friction $(^o)$; c (kPa) is the cohesion of soil; σ (kPa) is the applied normal stress and τ (kPa) is the corresponding shear stress.

According to OLS, the mathematical expression of $\beta = \tan(\phi)$ and of c are:

$$\beta = \tan(\phi) = \frac{\sum_{i=1}^{n} (\sigma_i - \bar{\sigma}) \cdot (\tau_i - \bar{\tau})}{\sum_{i=1}^{n} (\sigma_i - \bar{\sigma})^2} = r_R \cdot \sqrt{\frac{\sum_{i=1}^{n} (\tau_i - \bar{\tau})^2}{\sum_{i=1}^{n} (\sigma_i - \bar{\sigma})^2}} = \tag{2}$$

$$= r_R \cdot q$$

where n is the number of regression points $(\sigma_i, \tau_i),\ i = 1...n$ used in OLS procedure, r_R is the correlation coefficient assigned to the regression line:

$$r_R = \frac{\sum_{i=1}^{n} (\sigma_i - \bar{\sigma}) \cdot (\tau_i - \bar{\tau})}{\sqrt{\sum_{i=1}^{n} (\sigma_i - \bar{\sigma})^2 \cdot \sum_{i=1}^{n} (\tau_i - \bar{\tau})^2}} \tag{3}$$

and q is the spreading coefficient that is a measure of the dispersion values of τ_s related to the dispersion of σ_s values:

$$q = \sqrt{\frac{\sum_{i=1}^{n} (\tau_i - \bar{\tau})^2}{\sum_{i=1}^{n} (\sigma_i - \bar{\sigma})^2}} \tag{4}$$

The cohesion is estimated as:

$$c = \bar{\tau} - \bar{\sigma} \cdot \beta \tag{5}$$

where $\bar{\sigma}$ denotes the mean value of σ_i, $\ = 1...n$, used in OLS procedure; the same meaning applies for the upper line accent for τ, τ^2 and σ^2.

According to the classical approach, the uncertainty assigned to β is calculated as [6, 14, 15, 16, 35]:

$$u(\beta)^2 = \frac{1}{n-2} \cdot \frac{\sum_{i=1}^{n} \Delta_i^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \frac{S_o^2}{Q_{\sigma\sigma}} \tag{6}$$

where n is the number of points used in OLS regression, Δ_i is the difference between the measured value of τ_i and the calculated value for the i^{th} point of the regression line τ_{fi} i.e.

$$\Delta_i = \tau_{fi} - \sigma_i \cdot \beta - c \tag{7}$$

S_o is the constant uncertainty assigned to each τ_i:

$$S_o^2 = \frac{\sum_{i=1}^{n} \Delta_i^2}{n-2} \tag{8}$$

and

$$Q_{\sigma\sigma} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{9}$$

Based on eq. 6, the squared $u(\beta)$ can be expressed as:

$$u(\beta)^2 = \frac{1}{n-2}\beta^2 \left(\frac{1}{r_R^2} - 1\right) \tag{10}$$

The relative standard uncertainty of the slope, $u_R(\beta)$, estimated in the frame of OLS depends on n, but critically on r_R:

$$u_R(\beta) = \frac{u(\beta)}{\beta} = \frac{\sqrt{\frac{1}{r_R^2} - 1}}{\sqrt{n-2}} \tag{11}$$

One of the major drawbacks of the OLS approach is the critical dependency of $u_R(\beta)$ value on the r_R value, even so it depends on n value. Fig. 1 depicts the dependency of $u_R(\beta)$ on n value, but critically on r_R value. As could be observed in fig. 1, a value of $u_R(\beta) < 5\%$ can be achieved only if $r_R > 0.997$, whatever $n = 3...5$. For $r_R < 0.98$, $u_R(\beta)$ is greater than 10%, which is unacceptable for a professional DST measurement. Thus, eq. 11 and fig. 1 argue for increasing the number of regression points in DST practice, as long as the compromise between the decision risk and the economical reason is best.

In the case when $r_R \approx r_a = 0.9588$ and $n = 4$, as STN 73 1030:1988 specifies, a value of $u_R(\beta) > 20\%$ is expected, which is quite unacceptable for a good DST laboratory practice. Polish standard allows deviation of experimental point up to 25% related to the calculated one. These allowed deviations can decrease r_R below 0.95 and give rise to a value of $u_R(\beta) > 18\%$. ASTM D3080, ISO 17892-10:2018 and other current DST standards do not impose limits (conditions) for the deviations of the regression points, hence they allow for even larger $u_R(\beta)$.

The uncertainty of c, $u(c)$, is calculated based on 5 and on the so called *uncertainty propagation law* [35]:

$$u(c)^2 = \frac{\beta^2\left(\frac{1}{r_R^2} - 1\right)}{n-2}\left(\frac{Q_{\sigma\sigma}}{n} + \bar{\sigma}^2\right) = u(\beta)^2\bar{\sigma}^2 \tag{12}$$

The $u(c)$ value depends on the slope value, on the number of points n, on the average σ^2, on $Q_{\sigma\sigma}$, and again, it also depends critically on r_R. Thus, in the OLS

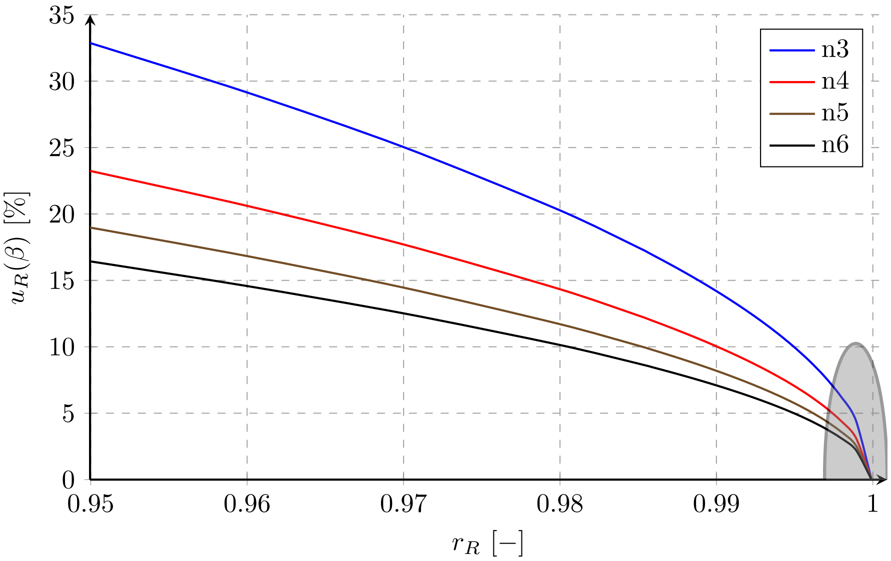


Figure 1: The $u_R(\beta)$ dependence on n and on r values

approach, the MU estimation of $u_R(\beta)$ values for the cases of fewer regression points, 3...5, provides reasonable relative uncertainties ($u_R(\beta) < 10\%$) only if $r_R > 0.99$, which means that, in such a case, the raw regression points lie on or are very close to the regression line.

2.2.2. IWLS method

The MU estimation of β, ϕ and c based on the classical approach can be easily criticized as it neglects some sources of uncertainties, such as measurements of σ_i, $i = 1...n$, and the covariance among (σ_i, σ_j), (τ_i, τ_j) for $i \neq j$. In this regard, IWLS was analyzed for estimating the $u(\beta)$ and $u(c)$ values. The IWLS was the first analyzed as it takes into account MU in both variables including their error correlations. The equations of IWLS, given by York et al. [33], were used to estimate β and c for the data set reported in [8]. The principle of IWLS consists in minimizing the sum:

$$S = \sum_{i=1}^{n} w_i \left(T_i - \beta\Sigma_i - c\right)^2 \tag{13}$$

Table 1: Symbols of the quantities used in IWLS approach, their formulae and the associated meanings [33]

Symbol	Formula	Meaning
$\bar{\Sigma}$	$\frac{\sum_{i=1}^{n} w_i \Sigma_i}{\sum_{i=1}^{n} w_i}$	Mean of the measured normal stresses
$\bar{T}$	$\frac{\sum_{i=1}^{n} w_i T_i}{\sum_{i=1}^{n} w_i}$	Mean of the measured shear stresses
U_i	$\Sigma_i - \bar{\Sigma}$	Reduced experimental normal stress
V_i	$T_i - \bar{T}$	Reduced experimental shear stress
γ_i	$w_i \left(\frac{U_i}{\omega(\Sigma_i)} + \frac{V_i}{\omega(T_i)} \right) -$ $- (\beta U_i + V_i) \frac{r_i}{\sqrt{\omega(\Sigma_i)\omega(T_i)}}$	Adjusting factor used to calculate σ_i, τ_i and β
β	$\frac{\sum_{i=1}^{n} w_i \gamma_i V_i}{\sum_{i=1}^{n} w_i \gamma_i U_i}$	Slope of the calibration line
c	$\bar{T} - \beta \bar{\Sigma}$	Intercept of the calibration line
σ_i	$\bar{\Sigma} + \gamma_i$	Adjusted normal stress
τ_i	$\bar{T} + \beta \gamma_i$	Adjusted experimental shear stress
$\bar{\sigma_i}$	$\frac{\sum_{i=1}^{n} w_i \sigma_i}{\sum_{i=1}^{n} w_i}$	Mean of adjusted normal stresses
$\bar{\tau_i}$	$\frac{\sum_{i=1}^{n} w_i \tau_i}{\sum_{i=1}^{n} w_i}$	Mean of adjusted experimental shear stresses
u_i	$\sigma_i - \bar{\sigma}$	Reduced adjusted normal stresses
v_i	$\tau_i - \bar{\tau}$	Reduced adjusted shear stresses
$u(\beta)$	$u(\beta)^2 = \left(\sum_{i=1}^{n} w_i u_i^2 \right)^{-1}$	Compound uncertainty assigned to the slope
$u(c)$	$u(c)^2 = \left(\sum_{i=1}^{n} w_i \right)^{-1} +$ $+ \bar{\sigma}^2 \cdot u(\beta)^2$	Compound uncertainty assigned to the intercept

where (Σ_i, T_i) are experimental values of the regression points, $i = 1...n$, and $w_i,\ i = 1...n$, are the weights of the points in the sum S, defined as:

$$w_i = \frac{\omega(\Sigma_i)\omega(T_i)}{\omega(\Sigma_i) + \beta^2 \omega(T_i) - 2\beta r_i \sqrt{\omega(\Sigma_i)\omega(T_i)}} \tag{14}$$

where $\omega(\Sigma_i) = u(\Sigma_i)^{-2}$; $\omega(T_i) = u(T_i)^{-2}$ and r_i is the correlation coefficient between errors in Σ_i and T_i.

The York‘s approach [33] makes use of the experimental point (Σ_i, T_i), and its corresponding point (σ_i, τ_i) that lies on the regression line, referred to as *adjusted point*. The estimation of the parameters provided by IWLS was performed based on the formulae given in tab. 1. Also, the symbols of the quantities and their significance are briefly given in tab. 1.

The IWSL approach was implemented according to the algorithm given by York et al. [33]. Thus, β is estimated by iteration and, subsequently, c, $u(\beta)$ and $u(c)$ were calculated using the quantities and formulae given in tab. 1.

2.2.3. WLOC method

The WLOC relies on the regression line $\tau = c + \beta \cdot \sigma$ that minimizes the sum of the products of the weighted vertical and horizontal differences between the measurement points and the line:

$$S = \sum_{i=1}^{n} \xi_i \left|\sigma_i - \sigma(\tau_i)\right| \cdot \nu_i \left|\tau_i - \tau(\sigma_i)\right| \tag{15}$$

where the $(\sigma_i, \tau_i),\ i = 1...n$are the measurement points; ξ_i and ν_i are the weights associated to σ_i and τ_i respectively.

The weights ξ_i and ν_i applied to each measurement point i are defined as $\xi_i = u(\sigma_i)^{-1}$ and $\nu_i = u(\tau_i)^{-1}$.

Eq. 15 can be written as:

$$S = \sum_{i=1}^{n} \xi_i \left|\sigma_i - \frac{\tau_i - c}{\beta}\right| \cdot \nu_i \left|\tau_i - \beta\sigma_i - c\right| = \sum_{i=1}^{n} \xi_i \nu_i \frac{(\tau_i - \beta\sigma_i - c)^2}{|\beta|} \tag{16}$$

which is similar to a weighted OLS approach.

The quantities used for calculation of the outputs of the regression line (β, c, $u(\beta)$, $u(c)$) based on WLOC approach, their associated formulae and meanings are given in tab. 2.

The minimization of errors in both X and Y directions are the major advantage of WLOC. The WLOC regression line passes through the centroid of the data $(\bar{x}, \bar{y})$, similar to the OLS and IWLS ones. WLOC provides analytical solution for β, which is an important advantage compared to IWLS that needs an iterative approach [26, 33, 34].

2.2.4. Orthogonal Distance Regression (ODR) method

ODR is applied in two cases i.e. with error in dependent variable (τ) and with errors in both variables e.g. (σ, τ). In the case of error in τ, ODR aims

Table 2: Symbols of the quantities used in WLOC approach, their formulae and the associated meanings [26]

Symbol	Formula	Meaning
$\bar{\sigma}$	$\frac{\sum_{i=1}^{n}\xi_i\nu_i\sigma_i}{\sum_{i=1}^{n}\xi_i\nu_i}$	Weighted mean of the measured normal stresses
$\bar{\tau}$	$\frac{\sum_{i=1}^{n}\xi_i\nu_i\tau_i}{\sum_{i=1}^{n}\xi_i\nu_i}$	Weighted mean of the measured shear stresses
$S(\sigma)$	$S(\sigma)^2 = \frac{\sum_{i=1}^{n}\xi_i\nu_i(\sigma_i-\bar{\sigma})^2}{\sum_{i=1}^{n}\xi_i\nu_i}$	Weighted standard deviation of the experimental normal stress
$S(\tau)$	$S(\tau)^2 = \frac{\sum_{i=1}^{n}\xi_i\nu_i(\tau_i-\bar{\tau})^2}{\sum_{i=1}^{n}\xi_i\nu_i}$	Weighted standard deviation of the experimental shear stress
Q	$\sum_{i=1}^{n}\xi_i\nu_i$	Sum of the point weights
β	$\frac{S(\sigma)}{S(\tau)}$	Slope of the calibration line
c	$\bar{\tau} - \beta\bar{\sigma}$	Intercept of the calibration line
$u(\beta)$	$u(\beta)^2 = \left(\frac{\partial\beta}{\partial\sigma_i}\right)^2\frac{1}{\xi_i^2} + + \left(\frac{\partial\beta}{\partial\tau_i}\right)^2\frac{1}{\nu_i^2}$	Combined uncertainty of the slope
$u(c)$	$u(c)^2 = \left(\frac{\partial c}{\partial\sigma_i}\right)^2\frac{1}{\xi_i^2} + + \left(\frac{\partial c}{\partial\tau_i}\right)^2\frac{1}{\nu_i^2}$	Combined uncertainty of the intercept
$c_i(\beta)$	$\frac{\partial\beta}{\partial\sigma_i} = \xi_i\nu_i\beta\frac{\bar{\sigma}-\sigma_i}{QS(\sigma)^2}$	Sensitivity coefficient of $u(\beta)$ related to σ_i
$\tilde{c}_i(\beta)$	$\frac{\partial\beta}{\partial\tau_i} = \xi_i\nu_i\beta\frac{\bar{\tau}-\tau_i}{QS(\tau)^2}$	Sensitivity coefficient of $u(\beta)$ related to τ_i
$c_i(c)$	$\frac{\partial c}{\partial\sigma_i} = -\frac{\xi_i\nu_i\beta}{Q}\left(1 + \frac{\bar{\sigma}(\sigma_i-\bar{\sigma})}{S(\sigma)^2}\right)$	Sensitivity coefficient of $u(c)$ related to σ_i
$\tilde{c}_i(c)$	$\frac{\partial c}{\partial\tau_i} = \frac{\xi_i\nu_i}{Q}\left(1 - \frac{\bar{\sigma}(\tau_i-\bar{\tau})}{S(c)^2}\right)$	Sensitivity coefficient of $u(c)$ related to τ_i

to find the parameters of the best calibration line minimizing the sum of the squared orthogonal offsets $S(c, \beta)$ [45] i.e.

$$S(c, \beta) = \sum_{i=1}^{n} \left(\frac{\beta \sigma_i + c - \tau_i}{\sqrt{1 + \beta^2}} \right)^2 \tag{17}$$

where $(\sigma_i, \tau_i),\ i = 1...n$are observed points.

The β is derived by minimizing $S(c, \beta)$ in with respect to c and β, which leads to:

$$b_{1,2} = B \pm \sqrt{1 + B^2} \tag{18}$$

where

$$B = \frac{q^2 - 1}{2qr} \tag{19}$$

where q and r have the previously mentioned significance. The cohesion, c, is calculated based on eq. 5. The $u(\beta)$ is derived as:

$$u(\beta) = \frac{S_o}{\sqrt{Q_{\sigma\sigma}}} \cdot \frac{\sqrt{(q^2 - 1)^2 + 4q^2r^2}}{2q^2r^2} \tag{20}$$

where $q,\ r,\ , S_o$ and $Q_{\sigma\sigma}$ have the aforementioned significance.

The $u(c)$ is calculated based on the uncertainty propagation law applied to eq. 5.

The main ODR approach with errors-in-both-variables is based on the hypothesis that σ_i variables, $i = 1...4$, are affected by normally distributed errors $N(0, u(\sigma_i))$, while τ_i is affected by $N(0, u(\tau_i))$ errors. Also, it assumes that $u^2(\tau_i)/u^2(\sigma_i) = \lambda = constant$ for any $i = 1...4$ [46]. The literature does not provide clear evidence to sustain the normal distribution of the errors affecting the DST measurands. The data addressed in the paper does not satisfy the assumption $u^2(\tau_i)/u^2(\sigma_i) = constant$, therefore only ODR with error in dependent variable was applied, aiming to compare the slope values given by OLS, ODR and WLOC when they are applied on the same DST data set. Hence, the ODR approach with errors-in-both-variables is not addressed herein.

2.3. HOLS method

2.3.1. HOLS principle

The HOLS is an improved version of the classical OLS i.e. the OLS regression is kept unchanged, but the way in which the uncertainties of β and c are estimated is changed. Thus, the HOLS approach takes into account the uncertainties of all input quantities i.e. $u(\sigma_i), u(\tau_i)$, including the covariance among them. The previous works of Nguyen et al. [7, 8, 9] have opened the way in this direction, but HOLS addresses all the error correlation coefficients of the input quantities. The HOLS assumes that the standard uncertainties of the input data and their error correlation coefficients are a priori evaluated i.e. $u(\sigma_i),\ u(\tau_i),\ r(\sigma_i,\sigma_j),\ r(\sigma_i,\tau_j)$ and $r(\tau_i,\tau_j),\ i \neq j,\ i = 1...n,\ j = 1...n$ are known. Thus, based on the uncertainty propagation law, the uncertainty of the slope can be expressed as [35, 39]:

$$\begin{aligned} u(\beta)^2 =& \sum_{i=1}^{n} \left(c(\sigma_i)u(\sigma_i)\right)^2 + \sum_{i=1}^{n} \left(c(\tau_i)u(\tau_i)\right)^2 + \\ &+ 2 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c(\sigma_i)c(\sigma_j)r(\sigma_i,\sigma_j)u(\sigma_i)u(\sigma_j) + \\ &+ 2 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c(\tau_i)c(\tau_j)r(\tau_i,\tau_j)u(\tau_i)u(\tau_j) + \\ &+ 2 \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c(\sigma_i)c(\tau_j)r(\sigma_i,\tau_j)u(\sigma_i)u(\tau_j) \end{aligned} \tag{21}$$

where $c(\sigma_i) = \frac{\partial\beta}{\partial\sigma_i}$ is the sensitivity coefficient that expresses the rate of β variation when σ_i varies; ibid $c(\tau_i) = \frac{\partial\beta}{\partial\tau_i}$; $r(\sigma_i,\sigma_j),\ i = 1...n,\ j = 1...n$, are the correlation coefficients assigned to the measurands denoted in the parentheses, that express the inter-dependency between σ_i and σ_j values. The same meaning stands for $r(\sigma_i,\tau_j)$ and $r(\tau_i,\tau_j)$, $i \neq j$. The use of eq. 21 ensures a complete accounting of the slope uncertainty, provided a proper assessment of the sensitivity and correlation coefficients has been performed.

The value of a sensitivity coefficient is a measure of the contribution of an influence factor to the uncertainty budget of the measurand as it multiplies the uncertainty of an individual contributor. Thus, the exact evaluation of the sensitivity coefficients of the uncertainty budget of the slope is a must. In the case of $u(\beta)$ budget, there are two sets of sensitivity coefficients, those related to $\sigma_i,\ i = 1...n$, denoted $c(\sigma_i)$, and those related to $\tau_i, i = 1...n$, denoted $c(\tau_i)$. The mathematical expressions of the sensitivity coefficients assigned to β were derived as:

$$\begin{aligned} c(\sigma_i) &= \frac{\partial \beta}{\partial \sigma_i} = \frac{\partial}{\partial \sigma_i}\left(\frac{\sum_{j=1}^{n}\left(\sigma_j-\bar{\sigma}\right)\left(\tau_j-\bar{\tau}\right)}{\sum_{j=1}^{n}\left(\sigma_j-\bar{\sigma}\right)^2}\right) \\ &= \frac{\tau_i-\bar{\tau}}{Q_{\sigma\sigma}} - 2\cdot\frac{Q_{\sigma\tau}}{Q_{\sigma\sigma}}\cdot\frac{\sigma_i-\bar{\sigma}}{Q_{\sigma\sigma}} = \frac{\tau_i-\bar{\tau}-2\beta(\sigma_i-\bar{\sigma})}{Q_{\sigma\sigma}} \end{aligned} \tag{22}$$

$$c(\tau_i) = \frac{\partial \beta}{\partial \tau_i} = \frac{\partial}{\partial \tau_i}\left(\frac{\sum_{j=1}^{n}\left(\sigma_j-\bar{\sigma}\right)\left(\tau_j-\bar{\tau}\right)}{\sum_{j=1}^{n}\left(\sigma_j-\bar{\sigma}\right)^2}\right) = \frac{\sigma_i-\bar{\sigma}}{Q_{\sigma\sigma}} \tag{23}$$

The sensitivity coefficients assigned to c, denoted $\tilde{c}(\sigma_i)$, were calculated as:

$$\tilde{c}(\sigma_i) = \frac{\partial c}{\partial \sigma_i} = \frac{\partial\left(\bar{\tau}-\bar{\sigma}\cdot\beta\right)}{\partial \sigma_i} = 0 - \frac{1}{n}\beta - \bar{\sigma}\frac{\tau_i-\bar{\tau}-2\beta\left(\sigma_i-\bar{\sigma}\right)}{Q_{\sigma\sigma}} \tag{24}$$

while $\tilde{c}(\tau_i)$ were derived as:

$$\tilde{c}(\tau_i) = \frac{\partial c}{\partial \tau_i} = \frac{\partial\left(\bar{\tau}-\bar{\sigma}\cdot\beta\right)}{\partial \tau_i} = \frac{1}{n} - \bar{\sigma}\frac{\sigma_i-\bar{\sigma}}{Q_{\sigma\sigma}} \tag{25}$$

The proper assessment of the correlation coefficients $r(\sigma_i, \sigma_j)$ for $i \neq j,\ r(\tau_i, \tau_j)$ for $i \neq j$ and for $r(\sigma_i, \tau_j)$ is the key problem of the adequate estimation of the $u(\beta)$ and $u(c)$ values. The available literature on this topic does not provide clear clues about the correlations of the errors of the DST measurands during the measurement process. The EUROLAB Technical Report 1 [35] indicates that the main correlation come from the same calibration used in σ and τ measurements. Another correlating factor in DST is the shear box.

A significant value of $r(\sigma_i, \sigma_j)$ implies a relatively strong correlation among the ways in which the influence factors behave. An experimental assessment of the $r(\sigma_i, \sigma_j)$ for DST is very costly, as it implies over 30 experiments. The assessment of $r(\tau_i, \tau_j)$ and $r(\sigma_i, \tau_j)$ based on experimental data is unrealistic, both from economic reasons and the impossibility to ensure identical specimens

Table 3: The matrix of the error correlation coefficients

**	σ_1	σ_2	σ_3	σ_4	τ_1	τ_2	τ_3	τ_4
σ_1	1	s	s	s	w	w	w	w
σ_2	s	1	s	s	w	w	w	w
σ_3	s	s	1	s	w	w	w	w
σ_4	s	s	s	1	w	w	w	w
τ_1	w	w	w	w	1	t	t	t
τ_2	w	w	w	w	t	1	t	t
τ_3	w	w	w	w	t	t	1	t
τ_4	w	w	w	w	t	t	t	1

for repetitive tests. In this context, the theoretical study of the dependency of the slope uncertainty on $r(\sigma_i, \sigma_j)$, $r(\tau_i, \tau_j)$ and $r(\sigma_i, \tau_j)$ seems to be a less costly way towards solving this difficult issue.

In the DST practice, the measurements of the normal force (N) and of the shear force (T) are done with separate devices, hence it is assumed that the $r(\sigma_i, \tau_j)$ have smaller values compared to that of $r(\sigma_i, \sigma_j)$ and $r(\tau_i, \tau_j)$ for $i \neq j$. Accordingly, the following correlation matrix is proposed were s stands for all $r(\sigma_i, \sigma_j)$, t for $r(\tau_i, \tau_j)$ and w for $r(\sigma_i, \tau_j)$ (tab. 3).

The worst-case strategy is another way to solve the problem of $r(\sigma_i, \sigma_j)$, $r(\tau_i, \tau_j)$ and $r(\sigma_i, \tau_j)$ values [35]. In this case, the MUs for different $r(\sigma_i, \sigma_j)$, $r(\tau_i, \tau_j)$ and $r(\sigma_i, \tau_j)$ values are evaluated and the greatest value of MU coming out from these calculations is considered as the representative value of MU.

2.3.2. HOLS algorithm implementation

The matrix approaches given in [34, 35] have suggested a matrix-based algorithm for the calculation of $u(\beta)$. This calculation procedure was tailored for complete assessment of the uncertainties of the DST outcomes. This matrix procedure comprises 6 steps:

(1) Calculation of the primary uncertainty matrix $u(x_i) \times u(x_j)$, denoted PUM, where x_i stand for σ_i and τ_i measurands, $i = 1...n$

(2) Calculation of the sensitivity coefficients $c(x_i) \times c(x_j)$ matrix, denoted SCM;

(3) Setting up the error correlation $r(x_i, x_j)$ matrix, denoted ECM;

(4) Calculation of the whole uncertainty matrix (WUM) by multiplying the PUM, SCM and ECM matrices term by term i.e.

$$a_{ij}(WUM) = a_{ij}(PUM) \circ a_{ij}(SCM) \circ a_{ij}(ECM) \tag{26}$$

where $a_ij()$ are the elements of a matrix lying on the line i and column j

(5) Summing the elements of the WUM matrix i.e.

$$u_c^2(\beta) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}(WUM) \tag{27}$$

(6) Estimation of the expanded uncertainty

The expanded uncertainty U with a given confidence level (usually 95%) is estimated based on a probability density distribution function (PDF) attributed to the measurand [18, 43, 44, 47, 48]. The assignment of a PDF to a measurand based on a Pearson test is a difficult task as it involves at least 100 measurements in repetitive or reproductive conditions [48, 49, 50]. On the other hand, considering that each measurand involved in linear regression is affected by at least 8 influence factors (soil type - fine-grained soil consistency index or coarse-grained soils density index - load speed, temperature, degree of consolidation, vibrations, equipment, human factors [3, 5, 6, 7, 8, 9, 10]), then the slope is affected by at least 64 factors. According to the Central Limit Theorem [50], a measurand whose uncertainty budget comprises more than 30 factors has a normal PDF. Hence, the PDF of the slope under consideration can be considered of the Gauss-Laplace type i.e. $N(\beta, u(\beta))$. These considerations argue for a coverage factor $k = 2$ to be used for the calculation of the expanded uncertainty of the slope with a 95% confidence level i.e.

$$U_\beta(95\%) = 2 \cdot u(\beta) \tag{28}$$

The uncertainty of ϕ and $u(\phi)$ can be calculated based on eq. 27, but one must take into account that the uncertainty range of ϕ is asymmetric. If one notes $\beta_o = \tan(\phi_o)$, then β belongs to the $[\beta_o - u(\beta);\ \beta_o + u(\beta)]$ interval. Consequently, ϕ varies in the $[\mathrm{atan}(\beta_o - u(\beta));\ \mathrm{atan}(\beta_o + u(\beta))]$ range. Thus, the two components of $u(\phi)$ can be differentiated i.e.

$$u_-(\phi) = \mathrm{atan}(\beta_o) - \mathrm{atan}(\beta_o - u(\beta)) \tag{29}$$

which is left-hand side of $u_c(\phi)$, and

$$u_+(\phi) = \mathrm{atan}(\beta_o + u(\beta)) - \mathrm{atan}(\beta_o) \tag{30}$$

which is the right-hand side of $u(\phi)$.

The expanded uncertainty of ϕ should be split in the right hand interval $U_{\phi+}(95\%) = 2 \cdot (\mathrm{atan}(\beta_o + u(\beta)) - \phi_o)$ and the left hand interval $U_{\phi-}(95\%) = 2 \cdot (\phi_o - \mathrm{atan}(\beta_o - u(\beta)))$ that are adjoined through ϕ_o.

The algorithm for the assessment of the expanded uncertainty of the intercept is similar to that used for the slope except for the values of the sensitivity coefficients. The PUM and ECM matrices used for uncertainty assessment of the slope are used in the calculation of uncertainty of the intercept c. Only SCM matrix has to be constructed for the calculation of the WUM matrix associated to the intercept. The expanded uncertainty of the intercept is calculated using a coverage factor $k = 2$, based on the same considerations made in the case of the expanded uncertainty of the slope.

3. Results and discussion

3.1. Comparative analyses of results provided by OLS, IWSL, WLOC, GNA, ODR and HOLS approaches

For a comparative analysis of the performance of the OLS, IWSL, WLOC, GNA, ODR and HOLS approaches we have chosen a data set obtained in the DST measurements carried on a CH soil specimen without fibers, marked as CH_0. These tests were performed in the Laboratory of the Department of Geotechnics, Faculty of Civil Engineering, University of Zilina, Slovakia [8]. The raw data published in [8] is given in tab. 4.

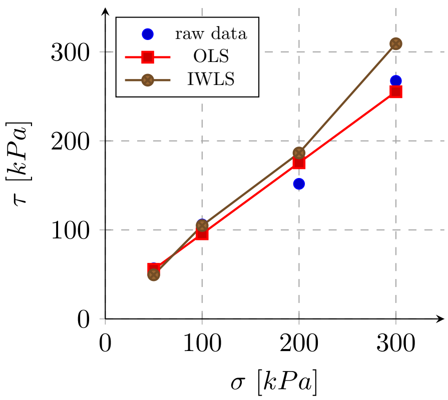
Table 4: Raw data published by G. Nguyen et al. [8]

Normal stress (σ)	Units $[kPa]$	MU $u(\sigma_i)$	Shear stress (τ)	Units $[kPa]$	MU $u(\tau_i)$
σ_1	50.0	0.1580	τ_1	56.8	0.3384
σ_2	100.0	0.3160	τ_2	106.1	0.6321
σ_3	200.0	0.6321	τ_3	151.7	0.9038
σ_4	300.0	0.9481	τ_4	267.4	1.5931
$< \sigma >=$	162.5		$< \tau >=$	145.5	

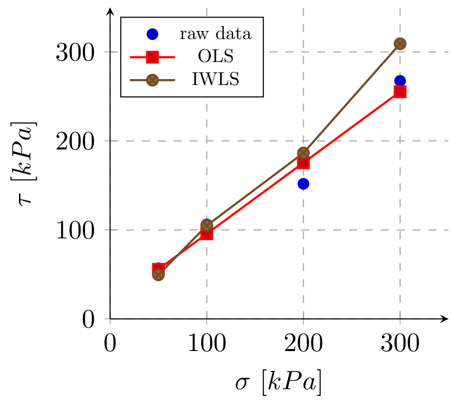
The regression lines obtained by OLS and by IWLS, for different error correlations coefficient values are shown in fig. 2.

As can be observed in fig. 2, the IWLS "adjusted points" are located much further away from the experimental ones compared to the OLS ones. Even worse, the shape of the IWSL regression line is curved and its curvature strongly depends on the r_{ii} values. The centroid coordinates given by IWLS (tab. 5) are located in the lower part of the DST regression line, which is another drawback of IWLS, because a linear regression performs better as the centroid lies in the middle of the data, as is the case of OLS [52, 53]. The β values obtained by OLS differ slightly from those obtained by IWLS for $r \leq 0$, but significantly from those with $r > 0.5$ (tab. 5). The c values obtained by IWLS are significantly higher than those obtained by OLS. Also, c monotonically increases as r increases.

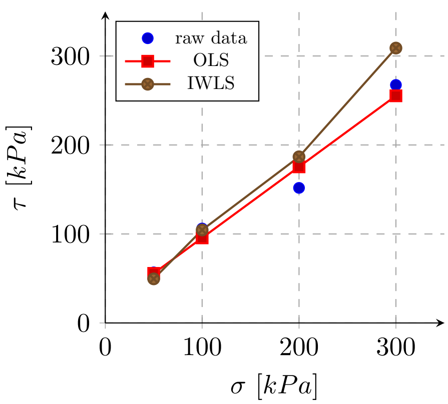
The OLS gives the greatest and the most unrealistic values for $u(\beta)$ and $u(c)$ compared to the other ones estimated based on GNA, HOLS and IWLS approaches. Apparently, the power of IWSL consists in providing lower uncertainties for both β and c parameters, at least 10 times smaller when comparing to those given by GNA and HOLS. But, this fact is a matter of computation and not a real finding. The DST practice shows that the uncertainties given by IWLS (tab. 5) are unrealistic [7, 8, 9, 10, 11]. Accordingly, the above findings lead us to consider the IWLS method as inappropriate to be applied in DST



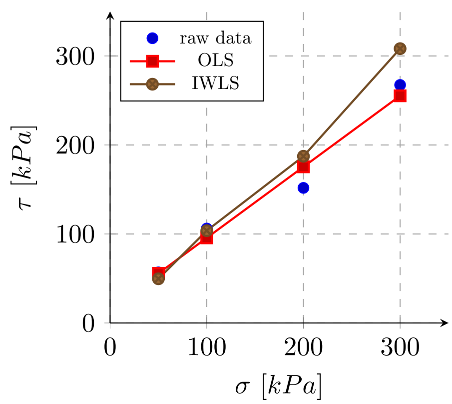
(a) $r = -1.0$



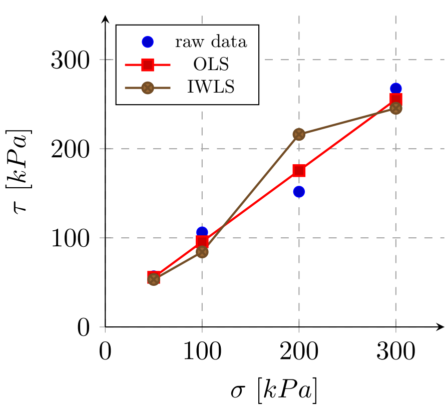
(b) $r = -0.95$



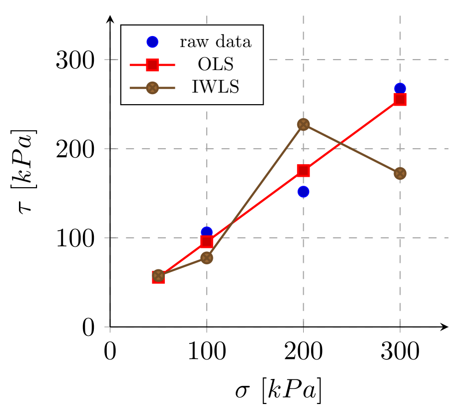
(c) $r = -0.5$



(d) $r = 0.0$



(e) $r = 0.95$



(f) $r = 1.0$

Figure 2: Regression lines obtained by OLS and by IWLS, for different error correlation coefficient values using the CH_0 specimen data [8]

Table 5: Comparative values of the β, c, $u(\beta)$ and $u(c)$ obtained with OLS, GNA, HOLS and IWLS approaches [8]

	OLS	GNA	HOLS	IWLS (r_{ii})						
				-1.0	-0.95	-0.5	0.0	0.5	0.95	1.0
β	0.798	0.798	0.798	0.773	0.774	0.770	0.772	0.764	0.693	0.637
c	15.84	15.84	15.84	19.23	19.24	19.24	19.42	19.78	24.96	29.92
$u(\beta)$	0.105	0.015	0.014	0.0022	0.0022	0.0019	0.0016	0.0012	0.0005	0.0004
$u(c)$	19.81	0.77	0.68	0.211	0.208	0.184	0.152	0.112	0.061	0.058
$< \sigma >$	162.5	162.5	162.5	76.84	76.86	77.11	77.67	79.29	97.92	111.93
$< \tau >$	145.55	145.55	145.55	78.64	78.65	78.84	79.23	80.39	92.75	101.31

Table 6: Comparative values of the β, c, $u(\beta)$ and $u(c)$ obtained with GNA [8], OLS, WLOC, IWLS and ODR approaches

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

practice, while HOLS gives reliable results.

As the IWSL has been proven unfit for DST processing , the WLOC method was tested on the same data, as to compare its results to the known values. The formulae given in 2 were used to evaluate the WLOC regression parameters. The WLOC we used [26] does not account for error correlation, therefore it can be considered similar to orthogonal distance regression (ODR), also known as Deming regression method [53]. The comparative results provided by WLOC, ODR and OLS regression approaches are given in fig. 3 and table 6.

The OLS, WLOC and ODR approaches provided similar calibration lines (fig. 3). This finding is supported by Mandel's relationship [54]:

$$\beta_{ODR} = \frac{\beta_{OLS}}{1 - \frac{s^2_{\sigma_i}}{s^2_{ex\sigma}}} \quad (31)$$

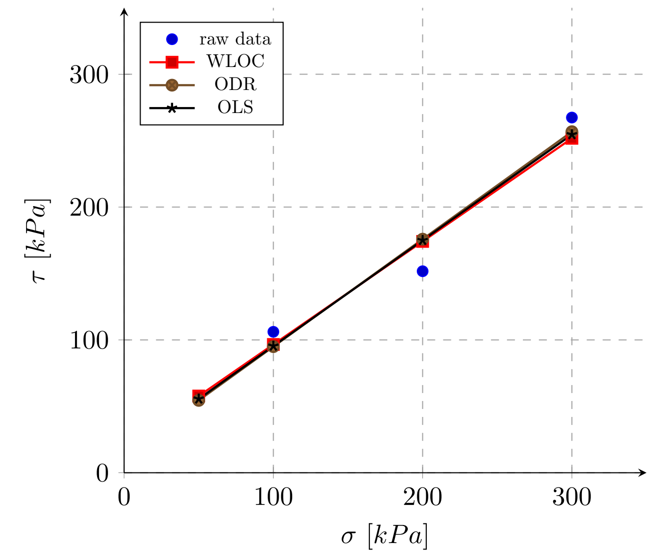


Figure 3: Comparative regression lines obtained by WLOC, ODR and OLS approaches

where β_{ODR} is the slope estimated by ODR approach, β_{OLS} is the slope estimated by OLS approach, $s^2_{\sigma_i}$ is the variance of a single σ, (σ_i), and $s^2_{ex\sigma}$ is the variance of the σ used for regression procedure.

According to eq. 31, the values of β_{ODR} and β_{OLS} are very close as in DST practice, the variance of a single σ_i value divided by the variance of the σ $(Q_{\sigma\sigma}/(n-1))$ is less than 10^{-5} for $i = 1...4$.

The c values given by WLOC and IWLS differ significantly from that provided by OLS, GNA and HOLS. Thus, c values given by WLOC and IWLS can be considered outliers. The WLOC approach behaves similar to IWLS one, given much smaller uncertainties than OLS. By comparing the data in tab. 6, and based on fig. 3, we considered the HOLS as being able to provide reliable values for β and c, as well as the WLOC, ODR and IWLS ones. On the other hand, the WLOC and IWLS methods failed in assessing reliable values for $u(\beta)$ and $u(c)$. Also, IWLS and WLOC provided unrealistic centroids i.e. $(77.67, 79.23)$ $[kPa]$, (IWLS for $r_{ii} = 0$), $(75.97, 77.95)$ $[kPa]$, (WLOC). These centroids lay far away from arithmetic centroid $(162.5, 145.5)$ $[kPa]$, nearby the lower left side of the calibration line. In such cases, any variation in ϕ cause the points to shift upwards from the true calibration line. Therefore, WLOC is considered unsuitable for MU estimation in DST, while HOLS provides reasons to be further tested by comparison with GNA for its validation.

3.2. Comparative analysis of GNA and HOLS outputs

The first comparison for HOLS validation used a piece of data published by Nguyen in [7]. The N and T values were measured using calibrated force transducers. The relative expanded uncertainty of the force transducer used for N measurement, given for a coverage factor $k = 2$, U_{RN} (95%), takes values in the $[0.12\%, 0.47\%]$ range, depending on the magnitude of measured force. For the sake of risk minimization, it has been taken into consideration a maximum value of the expanded uncertainty, i.e. $u_{RN}(\sigma) = 0.235\%$. The relative standard uncertainties of the shear box dimension were estimated based on repeated measurements and were found to have equal values $u_{Ra} = u_{Rb} = 0.20\%$. The calibration protocol of the force transducer of the shear force specifies $URT(95\%)$ in the $[0.12\%, 0.82\%]$ range. For the same reason, the maximum value was considered for the estimation of $u(\tau)$ i.e. $u_{RT} = 0.41\%$. Also, an uncertainty of type A ($u_A(\tau) = 0.5\%$) was considered to account for the specimen heterogeneity.

The normal stress σ is calculated as:

$$\sigma = \frac{N}{A} = \frac{N}{ab} \tag{32}$$

where N is the normal force in $[kN]$; and a, b are shear box dimensions in $[m]$.

In the uncertainty propagation approach and hypothesizing that no correlations occur among N, a and b, the uncertainty of σ can be expressed as:

$$u^2(\sigma) = \left(\frac{\partial\sigma}{\partial N}\right)^2 \cdot u_N^2 + \left(\frac{\partial\sigma}{\partial a}\right)^2 \cdot u_a^2 + \left(\frac{\partial\sigma}{\partial b}\right)^2 \cdot u_b^2 \tag{33}$$

where u_N, u_a and u_b are the standard uncertainties assigned to N, a and b, respectively.

According to eq. 33, $u(\sigma)$ can be expressed as:

$$u(\sigma) = \sigma\sqrt{u_{RN}^2 + u_{Ra}^2 + u_{Rb}^2} \tag{34}$$

Table 7

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

where u_{RN}, u_{Ra} and u_{Rb} are the relative standard uncertainties of type B of the N, a and b measurands.

The uncertainty assigned to τ measurand is estimated in the same manner as in the case of σ.

The heterogeneity of the tested soil is another contributor to the uncertainty budget of τ. This uncertainty was considered to be of A type and was estimated as relative standard uncertainty $u_{RA}(\tau) = 0.5\%$ [7]. Accordingly, the combined uncertainty $u_c(\tau)$ is calculated as:

$$u(\tau) = \tau\sqrt{u_{RB}^2 + u_{Ra}^2 + u_{Rb}^2 + u_{RA}^2} \tag{35}$$

The dependence of $u_c(\sigma)$ on σ and, also, that of $u_c(\tau)$ on τ imply an increase of MU as σ or/and τ increases, which determines a heteroscedastic behavior of the slope uncertainty [55].

Based on eq. 34, the value of $u_c(\sigma_{1-4})$ were recalculated and compared to those posted in [7] (tab. 7). The $u_c(\tau_{1-4})$ were recalculated based on eq. 35.

The exactness of the HOLS method depends, among others, on the accuracy of the values of the sensitivity coefficient. Hence, the values of the sensitivity coefficients were calculated based on the eqs. 22 - 25 and compared with those posted in [7]. Even though the derivatives of $c(x_i)$ are not posted explicitly in [7], the similarity between the recalculated values and the posted ones validates the exactness of the derivatives and the proper implementation of the computation algorithm in Excel.

The shear strength parameters calculated based on HOLS ($\beta = 0.45686$, $\phi =$

Table 8:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

22.545^o and $c = 2.11kPa$) are identical to those reported in [7]. The uncertainties of the calibration line calculated based on OLS are: $u(\beta) = 0.0101$, $u(\phi) = 0.48^o$ and $u(c) = 3.157kPa$. A better picture of these uncertainties is given by their relative uncertainties $u_R(\beta) \approx 2.2\%$, $u_R(\phi) \approx 2.2\%$ and $u_R(c) \approx 14.3\%$. Thus, OLS provides low quality of c as $u_{Rc}(c) \approx 14.3\%$. The GNA provided $u(\phi) = 0.33^o$, $u(c) = 1.26kPa$ [7] that are at least 50% lower than those given by OLS.

The algorithm described for the HOLS implementation was applied to estimate the uncertainties of the DST measurands based on the data given in tab. 7. Thus, the PUM matrix has been constructed based on the values of the standard uncertainties u(xi) given in tab. 8. These values were validated by comparison to the published ones [7].

The CSM matrix was elaborated in the same way as the PUM one and the values of its elements are presented in 9. The values of the sensitivity coefficients were validated by comparison to the published ones 7 [7].

The values of the error correlation coefficients $r(\sigma_i, \sigma_j)$ for $i \neq j$, $r(\tau_i, \tau_j)$ for $i \neq j$ and for $r(\sigma_i, \tau_j)$ couldn't be exactly established, therefore the ECM matrix has 3 degrees of freedom $r_{\sigma\sigma}$, $r_{\tau\tau}$ and $r_{\sigma\tau}$ that can be adjusted based on the laboratory evidences and on the operator expertise. For this case study, the following values: $r_{\sigma\sigma} = 0.2$, $r_{\tau\tau} = 0.2$ and $r_{\sigma\tau} = 0.1$ have been considered, which means that a correlation of 20% has been taken into account for the measurements carried with the same device, whilst a value of 10% has been

Table 9:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

Table 10:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

considered for the measurements carried with different devices (tab. 10).

Based on eq. 26 and on the values given in tables 8 - 10, the WUM matrix was constructed, which contains all the components of the uncertainties that affect the slope exactness (tab. 11).

The compound uncertainty $u(\beta)$ was calculated as the square root of the sum of the WUM matrix elements and the obtained value was $u(\beta) = 0.00482$ which

Table 11:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

Table 12:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

is approximately 2 times smaller than that obtained based on OLS approach. The corresponding $u(\phi)$ value to $u(\beta) = 0.00482$ is $u(\phi) = 0.458^o$, which is about 1.5 times greater than that reported $u(\phi) = 0.33^o$ [7]. This difference can be ascribed to the improper values assigned to $r(\sigma_i, \tau_j),\ i \neq j$.

The standard uncertainty of the cohesion, $u(c)$, was estimated using the same algorithm as for $u(\beta)$, but using the CSM given in tab. 12, instead of that given in tab. 9.

The compound uncertainty $u(c)$ was calculated as the square root of the sum of the elements of the WUM assigned to c and the obtained value was $u(c) = 0.997kPa$ which is approximately 3 times smaller than that calculated based on the OLS approach ($u(c) = 3.156kPa$), but it is close to that reported in [7] ($u(c) = 1.26kPa$).

The above case of $r_{\sigma\sigma} = 0.2,\ r_{\tau\tau} = 0.2$ and $r_{\sigma\tau} = 0.1$ should be seen as a matter of exemplification (i.e. how the calculation algorithm works), but there is no pertinent evidence for the $r_{\sigma\sigma} = 0.2,\ r_{\tau\tau} = 0.2$ and $r_{\sigma\tau} = 0.1$ values. As a consequence, to avoid underestimating the $u(\beta)$ values, and implicitly the $u(\phi)$ value, an exploration of the dependency of $u(\phi)$ value on $r_{\sigma\sigma},\ r_{\tau\tau}$ and $r_{\sigma\tau}$ has been conducted as it is shown in tab. 13.

According to tab. 13, the $u(\phi)$ value decreases monotonically as the values of $r_{\sigma\sigma},\ r_{\tau\tau}$ and $r_{\sigma\tau}$ increase from -1 to +1. Any negative correlation in $r_{\sigma\sigma},\ r_{\tau\tau}$ and $r_{\sigma\tau}$ gives rise to a $u(\phi)$ value greater than that corresponding to $r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = 0$. The value $u(\phi) = 0.33^o$ [7] is close to that given in the

Table 13: E

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

case of $r_{\sigma\sigma} = r_{\tau\tau} = 0.7$ and $r_{\sigma\tau} = 0.5$. On the other hand, Gillard [55] had shown (eq. 36) that for positive β a negative covariance should be expected in DST.

$$cov(\sigma_i, \tau_i) = -\beta \cdot u(\sigma_i)^2 \tag{36}$$

where (σ_i, τ_i) is a point of the regression procedure ($i = 1...4$) and β is the estimated slope.

Eq. 36 allows us to estimate $r_{\sigma_i\tau_i}$ as:

$$r_{\sigma_i\tau_i} = \frac{-\beta \cdot u(\sigma_i)^2}{u(\sigma_i)u(\tau_i)} = -\beta \cdot \lambda_i \tag{37}$$

where $\lambda_i = u(\sigma_i)/u(\tau_i)$ is a known experimental positive constant.

The $cov(\sigma_i, \tau_j)$, for $i \neq j$, are very difficult to be quantified; the same is true in the case of $cov(\sigma_i, \sigma_j)$, for $i \neq j$. On the other hand, the DST measurements for i and j points, $i \neq j$, are done separately, therefore it is reasonable to assume that there is a negligible correlation between σ_i and σ_j, σ_i and τ_j and, τ_i and τ_j caused by measurement processes. In such a case, only $r_{\sigma_i\tau_i}$, $i = 1...4$, are negative and the others are null. If it is assumed that $r_{\sigma_i\tau_i}$ has a constant value independent on i, $i = 1...4$, and the others covariance are negligible, then the values of $u(\phi)$ (tab. 14) do not differ too much from those given in tab. 13.

The worst case strategy, based on above data, implies totally negative error correlations. The difference between considering a total correlation ($r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = -1$) and neglecting error correlation ($r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = 0$) consists in the underestimation of the $u(\phi)$ value with $\approx 28\%$ compared to the case

Table 14:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

$r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = -1$, and with $\approx 24\%$ compared to the case $r_{\sigma\sigma} = r_{\tau\tau} = 0$, $r_{\sigma_i\tau_j} = 0$, for $i \neq j$ and $r_{\sigma_i\tau_i} = -1$ for $i = 1...4$. The case where $r_{\sigma\sigma} = r_{\tau\tau} = 0$, $r_{\sigma_i\tau_j} = 0$, for $i \neq j$ and $r_{\sigma_i\tau_i} = -1$ for $i = 1...4$ yields an underestimation of the $u(\phi)$ value of $\approx 4\%$ compared with the $r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = -1$ case.

Adopting *The worst case strategy* and a coverage factor $k = 2$, the expanded uncertainty assigned to ϕ is $U_\phi(95\%) \approx 1.4^o$ i.e. $U_{R_\phi}(95\%) \approx 6\%$ which is a reasonable one.

The $u(c)$ values behave in the same way as $u(\phi)$, as its value reaches a maximum for the case where $r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = -1$ ($u(c) = 1.53kPa$), a minimum for $r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = +1$ ($u(c) = 0.56kPa$) and it takes a middle value for $r_{\sigma\sigma} = r_{\tau\tau} = r_{\sigma\tau} = 0$ ($u(c) = 1.09kPa$). The case where $r_{\sigma\sigma} = r_{\tau\tau} = 0$, $r_{\sigma_i\tau_j} = 0$, for $i \neq j$ and $r_{\sigma_i\tau_i} = -1$ for $i = 1...4$ give a maximum $u(c) = 1.44kPa$. Adopting *The worst case strategy* and a coverage factor $k = 2$, the expanded uncertainty assigned to c is $U_c(95\%) \approx 3.1kPa$ i.e. $U_{R_c}(95\%) \approx 14\%$ which is comparable to those reported in [10, 11].

The quality of ϕ and c results, provided by HOLS for the case of CH soil, can be considered of medium level, as $U_{R_\phi}(95\%) \approx 6\%$ while $U_{R_c}(95\%) \approx 14\%$.

The HOLS provided greater MU values of the DST measurands given in [7] compared to GNA approach, but similar $U_\phi(95\%)$ and two times smaller $U_c(95\%)$ compare to OLS (tab. 15).

Table 15 shows that HOLS yields the most reliable MUs on the same DST data.

Table 15:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

3.3. Comparative analysis of the DST data obtained on soil reinforces with Polyester Fibers [8]

This section aims to underline the importance of the proper estimation of MU when one has to compare DST outcomes in order to establish the best solution for soil reinforcement with polyester fibers. In this regard, some data published in [8] were re-evaluated, but mainly the uncertainties assigned to the DST outcomes carried on soil from Čaradice, Slovakia. This soil was reinforced with TEXZEM PES 200 polyester fibers from Bonar Geosynthetics of $70mm$ in length, by 0.5% and 1.0% (wt.). DST were carried out in accordance with the Slovakian standard [22] using SHEARMATIC 300 equipment (Wykeham Farrance, CONTROLS Group, Milan, Italy). The shear box size was $0.3 \times 0.3m$. Specimens with and without polyester fibers were tested at normal stresses: 50, 100, 200 and 300 kPa.

According to the calibration protocol, the value of the relative standard uncertainty of the force transducer of normal force u_{RN} is in the $[0.022\%, 0.141\%]$ range depending on the magnitude of the force measured. For the range of normal forces applied during the tests, a standard relative uncertainty of 0.141% was used. Also, the relative standard uncertainty of the shear force transducer, u_{RT}, was in the $[0.025\%, 0.163\%]$ range depending on the magnitude of the force measured. For the range of shear forces applied in the test, a standard uncertainty of $u_{B,\tau_i} = 0.158\%$ will be used. Supplementary, a relative uncertainty of type A ($u_{RA}(\tau) = 0.5\%$) was considered to account for the specimen

Table 16:

Approach / Parameter	OLS	GNA	HOLS	WLOC	IWLS $r_{ii} = 0.0$	ODR
β	0.798	0.798	0.798	0.776	0.770	0.809
c $[kPa]$	15.84	15.84	15.84	18.96	19.33	14.01
$u(\beta)$	0.105	0.015	0.014	0.002	0.0016	0.020
$u(c)$ $[kPa]$	19.81	0.77	0.68	0.16	0.15	10.67

heterogeneity.

The relative standard uncertainty of the shear box dimension was considered as $u_{R_a} = 0.2\%$ (for dimension a); a similar value $u_{R_b} = 0.2\%$ was used for dimension b.

The combined uncertainties of the $\sigma_i,\ i = 1...4$, were calculated using eq. 34 and the relative uncertainties assigned to normal stresses. The same procedure was used for the combined uncertainties of the $\tau_i,\ i = 1...4$, but based on eq. 35 . The reported data [8] and the re-evaluated one are given in tab. 16.

The expanded uncertainties of ϕ and of c with 95% confidence level ($k = 2$) calculated by HOLS were considered for 3 representative cases i.e. neglecting all error correlations, (denoted H00), the worst case for correlation of the individual measurement $r(\sigma_i, \tau_i) = -1,\ i = 1...4$, (H$\bar{1}$0), and the worst case for total correlation of errors (H$\bar{1}\bar{1}$). The idea behind this option relies on the evidences that H00 is frequently encountered in DST, H$\bar{1}$0 seems being the most realistic DST case, whilst H$\bar{1}\bar{1}$ is truly the worst case for MU in HOLS approach.

The analysis carried on the data published in [8] has shown that the reported values of the DST parameters (ϕ, c) for all 6 samples under consideration (tab. 16) are identical for ϕ and for c. The reported values of the expanded uncertainties of ϕs and cs [8] are underestimated by the GNA approach compared to the HOLS ones, especially for the H$\bar{1}$0 and H$\bar{1}\bar{1}$ cases. The GNA approach provides similar values for $U_{R_\phi}(95\%)$ and $U_{R_c}(95\%)$ compared to the HOLS ones for the H00 cases. These findings can be easily explained, considering that GNA makes use of positive error correlation coefficients. If *the worst case* is applied for MU estimation for the sake of risk mitigation in decision making i.e. H$\bar{1}\bar{1}$, then the cases of CS and CH specimens can be disjointed. Thus, the variations of the DST parameters of CS specimens can be supported as the difference of their values exceeds the sum of their assigned expanded uncertainties, but in the case of CH samples the variations cannot be stated, as the associated coverage intervals to ϕs overlap. This finding highlights the importance of the parsimonious estimation of MU when needed to assess the research progress (improvement rate of soil strength expected by the researches).

4. Conclusions

The uncertainty estimation of the DST results is of great demand for geotechnical practice, but the current DST standards do not address this issue. Furthermore, DST standards prescribe OLS, although both DST measurands, i.e. normal stress and shear stress, are affected by correlated uncertainties.

The paper demonstrates that OLS, IWLS and WLOC approaches are not suitable for estimating Mus in DST practics, as OLS largely overestimates MUs, while IWLS and WLOC strongly underestimate them. The HOLS is a holistic development of the GNA approach. Unlike GNA approach, HOLS takes into account all the contributions to the uncertainty budget of the DST. HOLS deals properly with homoscedastic data, but most importantly, with heteroscedastic DST data.

The algorithm for HOLS implementation relies on a matrix data processing, was developed in Excel and it was validated by inter-comparison between published data [7, 8] and data obtained with the HOLS approach. The proposed algorithm can be implemented easily in Excel following the steps presented in the paper, but also in other commercial software as MATLAB, Mathcad etc.

Further researches are needed for accurate estimation of the correlation amplitudes among the entry quantities (σ_i, τ_i) in DST. Combined experimental observations and Monte Carlo simulation are foreseen as one route to solve this issue. The other route can be the top-down approach which avoids assessing the individual contributions, but accounting them at bulk level [39, 56]. Until then, *the worst case strategy* remains the only way to mitigate the risk of using DST results in geotechnical activity.

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