Comparative numerical modelling of granular soil embankment stability using LEM, FEM, FEM-SPH and DEM approaches

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ABSTRACT: The current paper aims to highlight the limitations of different methods of safety factor assessment and display the failure mechanisms obtained through various techniques when modelling cohesionless soil embankments. In order to emphasize the advantages and disadvantages of each of the selected methods – Limit Equilibrium (LEM), Finite Element (FEM), Finite Element modified with Smoothed Particle Hydrodynamics (FEM-SPH) and Discrete Element (DEM), an embankment made of unreinforced rock is considered.

The second objective to be accomplished is determining which of the aforementioned methods may take into account the dual behaviour of compacted coarse material upon which important vertical stresses act – a low to medium cohesive material behaviour, with a large internal friction angle, due to granular interlocking, and a pure cohesionless material, once the instability phenomena appears.

KEYWORDS: granular soil, stability, LEM, FEM, SPH, DEM

1 INTRODUCTION

It is well known that in the case of coarse granular soils, only the internal friction angle offers the material's shear strength. Improved studies on their behaviour (Ning and Godth 2013), show that interlocking may lead to obtaining cohesion values large enough to be taken into account. Following the appearance of the failure phenomena, the material loses its apparent cohesion, and the continuum transforms into a sum of independent particles. This "phase" transformation may be taken into consideration only in the case of advanced models, leading to what is widely known as value engineering.

The present paper is debating all these computational possibilities applied to a road embankment of 20m height, having the geometry depicted in fig. 1.

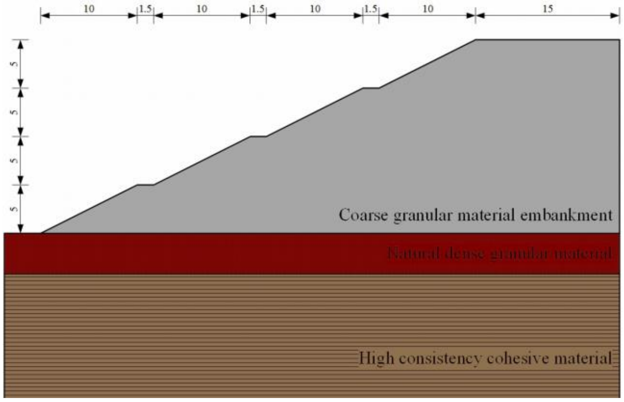

Figure 1. The analysed model geometry

At the upper part of the model, a 25kPa load has been taken into account, simulating the traffic and road structure laying at the top of the embankment.

The considered material parameters for the three soil types are as displayed in tab. 1.

Table 1. Parameters used in modelling

Soil	γ [kN/m^3]	ϕ [°]	c [kPa]	ψ [°]	E [kPa]	ν [-]	ε_t [%]
Type I	21	38.6	29.8	5	42000	0.25	15
Type II	19	25	10	0	10000	0.35	12
Type III	19	12	50	0	30000	0.28	-

* ε_t – strain threshold for the FEM-SPH model

1 LIMIT EQUILIBRIUM MODEL

One of the first historically dated methods for computing a slope's stability is the Limit Equilibrium Method (LEM), which takes into account the shear resistance parameters and the forces acting on different slices of soil: horizontal shearing/resistance force, own weight, later to be added parameters such as suction or damping. A representative method, nowadays automated by usage of mathematical algorithms and included into calculation software is the Bishop model (Bishop 1955).

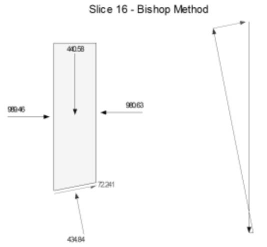

Figure 2. Typical slice forces diagram using the Bishop Method (horizontal soil resistance and thrust, own weight, tangential and normal forces)

The calculation in the case of Bishop model takes into account the soil's own weight, cohesion and internal friction angle.

Regarding the study case, an overall safety factor of 1.992 has been obtained, thus leading to a highly stable geometry versus the material's characteristics. In fig. 3, it may be noticed that the critical slip surface is covering almost a third of the embankment, advancing also through the foundation soil.

If the apparent cohesion is not taken into account, an overall safety factor of 1.910 is obtained. The geometry of the embankment and the good foundation soil provides high values of the safety factor, but nevertheless one should bear in mind the influence of the apparent cohesion when safety factors close to 1 are obtained.

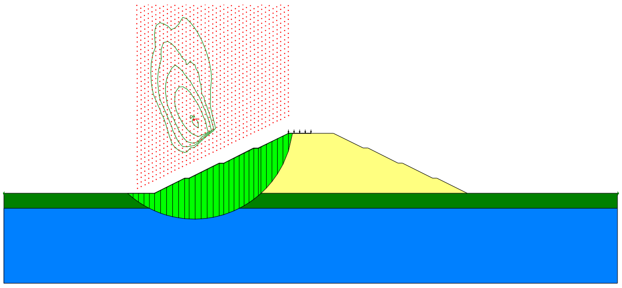

Figure 3. Critical slip surface of the embankment using Bishop Method

2 FINITE ELEMENT METHOD MODEL

The Finite Element Method (FEM) became the standard analysis method in engineering quickly after the computational power allowed the implementation of its solving algorithms, allowing complex phenomena to be modelled (Zienkiewicz, Taylor and Zhu 2005). In geotechnical engineering, it is one of the few methods that may take into account the hydro-mechanical coupling needed in the case of consolidation analyses or other degrees of freedom (thermal, electrical etc.) that may appear in complex problems.

Slope stability analysis using FEM shows multiple variations, starting from a general determination of stresses that may appear and determining if the calculations converged to a solution (stable slope), and continuing to shear resistance reduction method, where the internal friction angle and/or the cohesion are gradually diminished until a failure surface develops.

In the current problem, the general approach was considered. The obtained results, depicted in terms of plastic strains in fig. 4, show a critical slip surface resembling the one obtained using LEM, while offering a glimpse on the possible deformations that may appear. The overall safety factor computed manually through means of contour integration offered a value of 1.110.

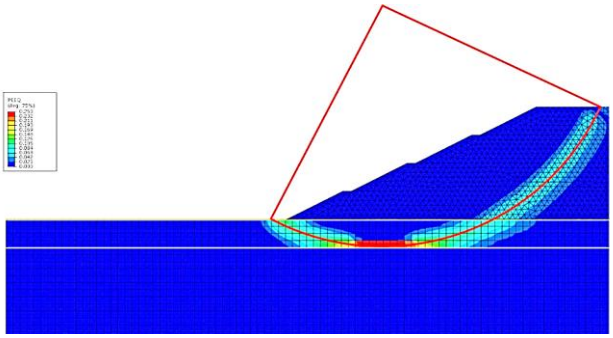

Figure 4. Critical slip surface of the embankment according Finite Element Method

The limitations of this method consist both in offering a variety of values regarding other possible failure surfaces that LEM is able to compute. Nonetheless, the information quantity in terms of stresses, deformations, strains, and critical points that is offering make the FEM a current choice in engineering practice.

3 FINITE ELEMENT METHOD USING SMOOTHED PARTICLE HYDRODYNAMICS

A newer extension of the Finite Element Method is represented by the Smoothed Particle Hydrodynamics (SPH). The method proposed by (Gingold and Monaghan 1977) is a meshless discretization of continuum partial differential equations, using an evolving interpolation scheme to approximate field variables at any point in the domain. In the case of small deformations, the method offers lower quality results than the classical Lagrange approach, while for large deformations, a coupled Euler-Lagrange model provides more accurate answers. Still, the method allows a low cost model to be built, in terms of computing power, offering a comprehensive view on the mechanism of the instability phenomena.

The model is first created as a continuum, with classical Lagrangian formulations of material properties and boundary conditions, while some parts of the model are tagged as possible triggers for SPH particles. The particles are created when a continuum element reaches a pre-established threshold in terms of stresses, strains or time. These newly generated elements are to be held inside the model using new boundary conditions (e.g. boxes), as the nodes on which the initial boundaries were applied disappear.

The method, still under discussion from the geotechnical engineering point of view, has been considered in modelling of impact of bullets, ice or concrete crushing (Jankowiak and Lodygowsky 2011; Delsart, Fabis and Vagnot 2011; Lescoe 2010) with promising results.

Concerning the proposed benchmark model, the results display the shape of the critical surface as depicted in fig. 5 and fig. 6. The cracks appearing into the continuum mass represent elements with distortion reaching a certain threshold of strain (considered as the strain corresponding to the peak shear strength values).

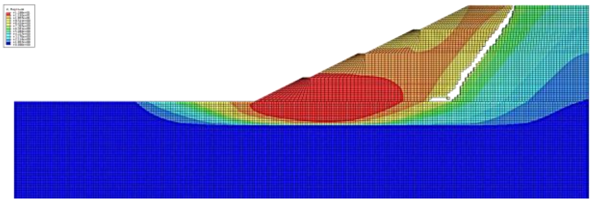

Figure 5. Critical slip surface of the embankment according FEM-SPH applied only to the embankment (particle display is turned off)

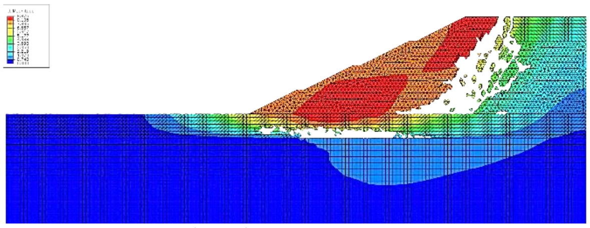

Figure 6. Critical slip surface of the embankment according FEM-SPH to both the embankment and the foundation soil (particle display is turned off)

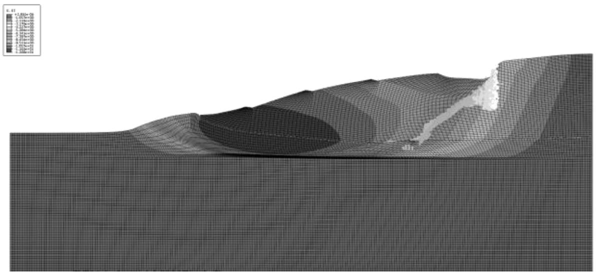

Figure 7. The sliding soil mass shape obtained through FEM-SPH simulation (only for the embankment)

The results display the continuum inner elements, the particles resulted from the transformation and the field variables of the two. In this case, the possibility of determining the overall safety factor using classical integration method is null, as the particles are free to move within the model, constrained only by the interaction with other particles or elements.

The end results (fig. 7) display the final, stabilized shape of the soil sliding mass, thus offering an idea on the stress-strain state during and after the sliding took place. However, only a qualitative assessment of the slope stability may be directly obtained, by means of SPH zones coverage.

4 DISCRETE ELEMENT METHOD MODEL

The Discrete Element Method (DEM), introduced by (Cundall 1971), is a powerful tool for modelling granular assemblies, since it handles the material properties at an element-based scale. The behaviour of the material as a whole is governed by the interaction between the particles that comprise it. Although the mathematical equations that stand behind DEM are not complex (they are not to be described herein, but many papers can be found on the subject, such as Luding 2008), the sheer number of computations needed performed makes the method difficult to apply on a large scale model. Therefore, keeping the number of particles relatively low is a requirement when performing the analysis.

For the current work, the simulations were performed using a software created by the authors (Priceputu 2013). A quasi-3D model was constructed (slightly modified from the initial geometry), with 2 dimensions much larger than the third – the in plane dimensions are about 70x40m, with a thickness of 1.0m. A cross-section of the boundaries of the model is shown in fig. 8. To produce the boundaries, two separate objects were created: a bottom container, and an embankment slope boundary, both of them being constructed from box shapes. The bottom container consists of 3 boxes describing the bottom, left and right edges of the model, and is always kept fixed during the simulations. Its associated material is the same as the one used for the particles. The other boundary is made of 10 frictionless boxes, describing the shape of the slope. Although the total effective height of the boundaries is 40m, they are only filled with particles up to a height of 30m, the rest of the area being used to generate particles.

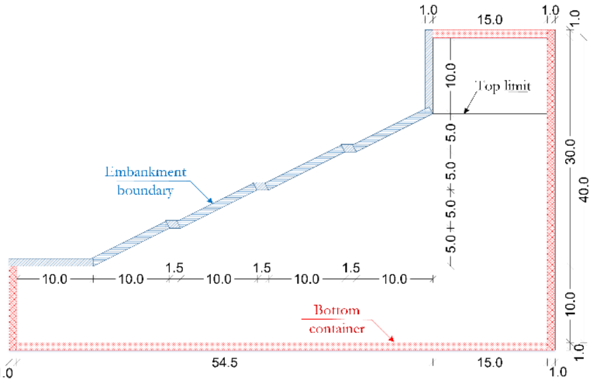

Figure 8. Cross-section through the model boundary elements.

The model consists of two stages: particles generation and free-slope analysis. During the first stage, the boundary is filled with particles created in groups of 1000, within the upper region, until the top limit is reached. In order to assure a proper filling of the bounded volume, the particles are kept frictionless until the filling stage is almost complete. When the free-slope analysis stage commences, the top boundary is released and the embankment is let to develop until an equilibrium is reached.

Several models were created with different particle shapes and sizes, as shown in tab. 2. The first model contains only spheres with diameter 0.5m, the second one contains cubes with the side length of 0.5m, while the third was developed creating both cubes and spheres, with equal probability for each one. The density of the material is 2.65g/cm^3, and the friction coefficient for the material interface is 1.0.

Table 2. Models configuration setup

Model number	Spheres	Cubes	Side / diameter [m]
1	14372	0	0.5
2	0	11689	0.5
3	6290	6141	0.5

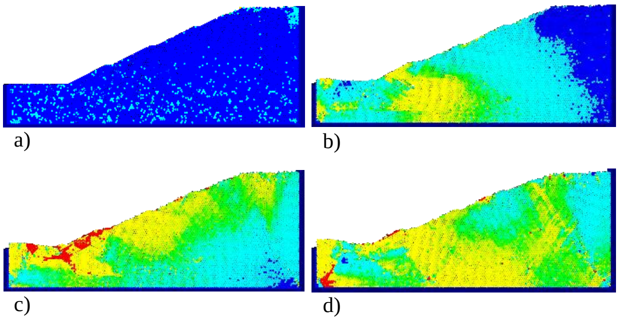
a) b) c) d)

Figure 9. Development of particle velocities in model 1 after a) 0.02s, b) 1.00s, c) 2.00s, d) 3.00s.

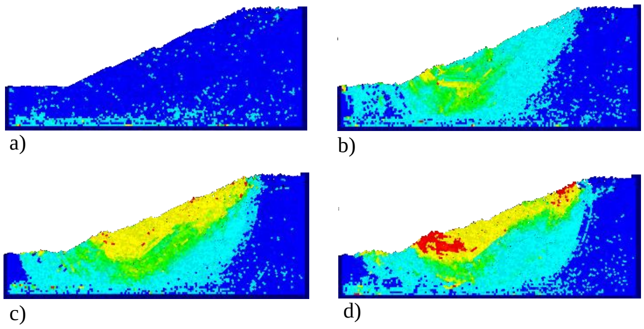
a) b) c) d)

Figure 10. Development of particle velocities in model 2 after a) 0.02s, b) 1.00s, c) 2.00s, d) 3.00s.

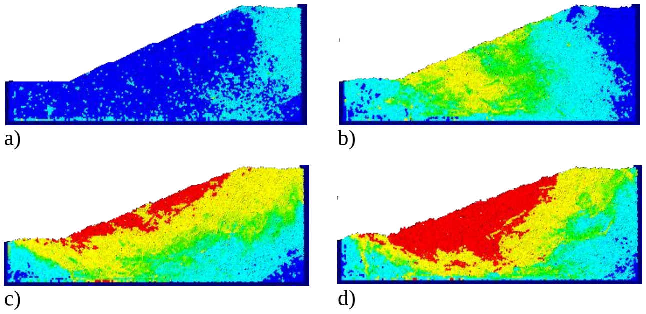
a) b) c) d)

Figure 11. Development of particle velocities in model 3 after a) 0.02s, b) 1.00s, c) 2.00s, d) 3.00s.

The first 3 seconds of the models are shown in terms of total velocities of each particle in fig. 7 to 11. Although the parameters used are the same for all models, the behaviour of the material is quite different, in terms of shape of the failure surface, rate of failure and final stable configurations. The first model (fig. 9), consisting of spheres only, tends to develop failure at a slower rate, and does not show any clear failure surfaces, but rather a general tendency of translation. The stability is gained after 13.8s, when the total kinetic energy of the system drops below 30kJ, while the maximum obtained was 596kJ. The second model (fig. 10), having particles of cubic shape, develops a circular failure surface at an early stage, but the maximum velocity rarely exceeds 1.0m/s. It also reaches its final configuration after less than 8.0s, when the total kinetic energy is approximately 28kJ, while the maximum obtained was 393kJ.

Model number 3 (fig. 11) consisting of both spheres and cubes is the most dynamic and displays a very explicit circular failure surface. The total kinetic energy reaches a maximum of 1273kJ after 5.0s when more than 40% of the particles have velocities larger than 1.0m/s. The energy drops below 100kJ after 11.25s and reaches a minimum of 37kJ at the end of the 12.0s of simulation. The final obtained configuration renders an angle of repose less than 10°, much smaller than those obtained on the other two models, which produced local slopes of 34°.

5 CONCLUSIONS

A granular soil embankment model was analysed using a variety of numerical stability analysis techniques. The discrete element method is considered to best simulate the behaviour of the modelled material, since LEM does not take into account the deformations, while the FEM exhibits severe convergence problems when modelling purely cohesionless materials.

Although in the case of granular soils, the failure surface is considered to follow a plane shape, tilted at an angle equal to 45°+ϕ/2, according to (Coulomb 1776) theory, not all of the obtained results are consistent with this assumption. If the DEM models are considered, only pure spherical grains lead to a cohesionless-like behaviour, displaying a translational type of failure, while the other simulations depict a rotational type of sliding, with clear circular surface, usually associated to the presence of cohesion. This obtained apparent cohesion may be explained by interlocking of the particles which leads to an additional shearing strength against rolling failure mechanisms, which requires a larger energy than slipping failure. This effect may be macro-mechanically described by cohesion, although the micro-properties of the material, determined on small samples, are purely frictional.

The continuum models are created taking this apparent cohesion into account. Only when considering a FEM-SPH model (when SPH particles are generated only within the embankment) a quasi-plane failure surface is developed, which can be explained by either insufficient possibility of complete failure surface development, or the spherical nature of the SPH generated elements. When the first foundation soil (also granular material) is allowed to convert continuous elements into SPH, the failure mechanisms resembles the DEM model with mixed particle shapes, characteristic to a low to medium cohesive material.

The LEM model is not taken into account when discussing the shape of the failure surface, since the analysis was conducted assuming a circular type of failure. This method can only be employed to assess a degree of safety, much more valuable after analysing the behaviour of the model using more complex methods. The latter usually allow qualitative assessment of the safety factor, but offer a large volume of data regarding the stress-strain state of the model, and describe the failure mechanism, but are more difficult to compute a point-wise value of the safety factor.

The advantages and disadvantages for each of the considered methods can be assessed from different points of view, such as complexity of model formulation regarding coupled phenomena, computational requirements, ease of use and time consumption, reliability, experience and so on. Moreover, the necessary input data may have a decisive impact on choosing the approach, often being restricted by lack of information (the most common minimum data requirements are displayed in tab. 3).

However, in engineering practice, some situation, such as forensic back-analysis, impose the usage of more advanced techniques that offer a comprehensive view of the developing phenomena and calibration using the final (observed) configuration. A brief overview on the supplied information according to the discussed methods is presented in tab. 4.

Table 3. Parameters used in modelling

Model acronym	γ [kN/m^3]	ϕ [°]	c [kPa]	ψ [°]	E [kPa]	ν [-]	$\varepsilon_{threshold}$ [%]
LEM	✓	✓	✓	-	-	-	-
FEM	✓	✓	✓	✓	✓	✓	-
FEM-SPH	✓	✓	✓	✓	✓	✓	✓
DEM	✓	✓	-	-	-	-	-

Table 4. Obtained data

Model acronym Data	LEM	FEM	FEM-SPH	DEM
Safety factor	✓	✓	-	-
Critical failure surface	✓	✓	✓	✓
Failure mechanism	-	-	✓	✓
Evolution in time	-	-	✓	✓
Final state	-	-	✓	✓

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