COMPARISON BETWEEN STOCHASTIC AND DETERMINISTIC APPROACHES IN SLOPE STABILITY ANALYSIS

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ABSTRACT

In slope stability analyses, it is rather difficult to offer accurate solutions when using a deterministic approach, due to the uncertainties related to the soil's mechanical parameters. The deterministic methods are based almost entirely on the value of the well-known safety factor, as a measure of stability reserve, and do not take into consideration that many of the used parameters are random variables affected by uncertainties caused by different factors. In order to evaluate the effects that these uncertainties have on the process of finding a critical failure surface and its corresponding safety factor, it is necessary to employ probabilistic methods, which associate a degree of confidence to the obtained solution. This paper consists of a brief review of the methods and principles when using probabilistic methods to assess the stability of a slope. A simple example is used to compare the results of stochastic and deterministic methods, leading to a discussion on the advantages and disadvantages of these methods when used in common engineering practice.

Keywords: slope stability, limit equilibrium methods, finite element method, stochastic methods

INTRODUCTION

In the slope stability analysis historical evolution, an important factor was the computation capability of the phenomenon. A first approach started with the notion of stable slope developed simultaneously with the soil shearing strength theory, developed by Coulomb. The stable slope approach is a very good tool for pre-design stages of artificial slopes, such as later developments appeared, especially the contributions of Taylor [1].

The first proposed methods for computing the stability of slopes employed limit equilibrium methods (LEM), considering the mechanical equilibrium of forces, moments or stresses acting on an independent sliding body over a fixed mass. The most common limit equilibrium methods were the methods of slices ([2], [3], [4], [5], [6]), the friction circle [1] and the logarithmic spiral method [7]. The slices models were the first approaches to be employed in the computation of slope stability since they are fit for hand computation. These models assume that the sliding mass behaves as a set of incompressible elements (vertical slices) acting among them and with the rest of the mass after a set of equilibrium conditions (force, moment or stress balance). The failure surface is either set in shape (e.g. circular-cylindrical), case in which the critical surface is found using optimization algorithms (either empirical location [2] or by brute force, computing stability factors for a set of possible surfaces), or, for a given predefined failure surface, such as the case of a translational landslide over bedrock, the stability factor is computed. The former type of analyses may be considered as a first stochastic approach where it was found the probable failure surface as the one with the lowest associated safety factor.

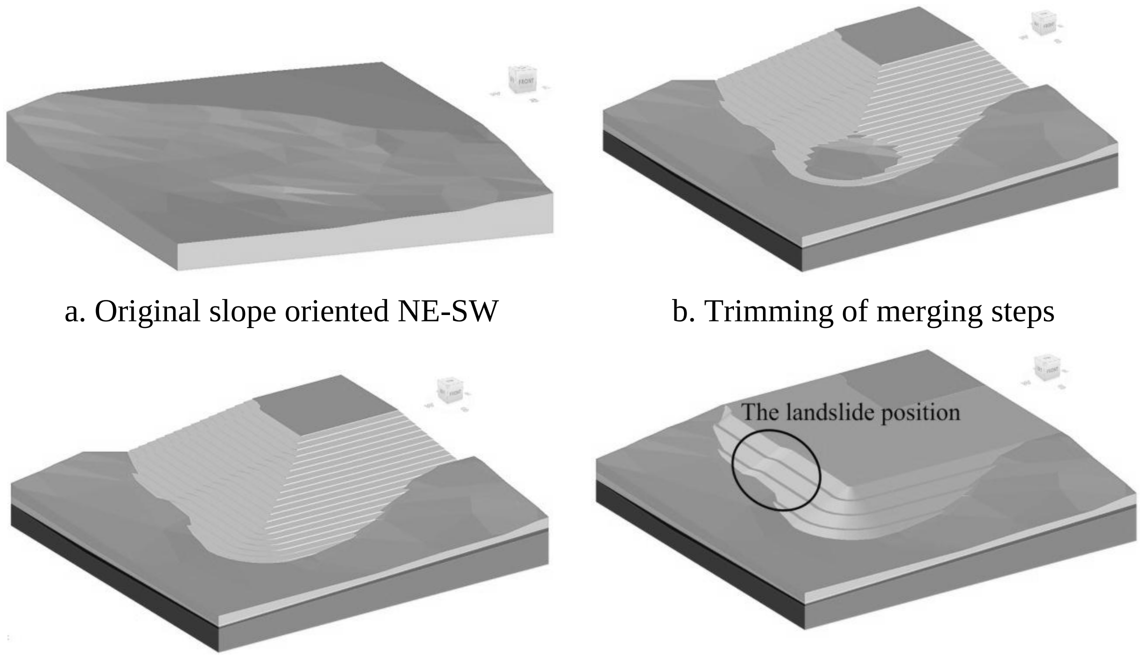
The Lagrangian formulation of finite element method (FEM) has been used since the geometric non-linearity concept was introduced in computation, because the basic FEM formulations, deriving from the variational computation principal (virtual work principal and effects superposition principal) did not allow, initially, high deformations. Nowadays, it may be noticed that adaptive limit equilibrium methods and two-dimensional finite element methods analyses are used in current practice, but the development of analyses approaches tends towards alternative methods which enable both the three-dimensional approach study, as well as the assessment of the propagation zones. These methods are Euler-Lagrange finite-element coupling or discrete element method (DEM), but, due to the specific mechanical parameters and/or the high requirement of computational capacity, they are still in their infancy from the engineering practice point of view.

A concerning matter is the accuracy of the results, mainly the difference between the computationally obtained and the actual safety factors. Several studies have been performed on this issue, concerning limit equilibrium methods, but disregarding the accuracy of the employed soil parameters ([8], [9], [10], [11], [12], [13], [14]). Duncan [15] concluded that regardless of the computational method, the obtained factor of safety is virtually the same for similar problems. He noted that the differences between the safety factors for the methods of slices analyses, friction circle analyses, logarithmic spiral analyses or finite-element analyses, lie below 12%.

In our opinion, confirmed in several case histories, one of which being presented herein, the hypothesis of the limit equilibrium methods leave non-conservative issues which contribute to higher stability factors than the actual phenomenon. The sliding mass is incompressible and regarded as an independent moving body acting with the same displacement rate in all the points on the failure surface. Indeed, regardless how the equilibrium equations are written with respect to the mechanical system, the results are just slightly influenced by various factors such as slices interaction, but more significantly influenced by pore-water effect on weight and/or shearing strength parameters.

CASE HISTORY

In order to illustrate the importance of correct choice of parameters and numerical analysis method, it shall be discussed a case history regarding a 28m tall embankment, filled in over an existing slope (Figure 1). The embankment was part of a transportation infrastructure which, after completion, was in service for about two years before a landslide occurred on a side. The direction of the natural slope was descending from NE to SW. The inclined surface had a concavity, reason for which, the merging between the natural ground and the infill material has been done in steps covered with a drainage layer bounded by geotextiles, in the shape of steps. Eventually, the additional material (cohesive) was compacted after being excavated from a borrow pit on the site.



a. Original slope oriented NE-SW

b. Trimming of merging steps

c. Infilling merging steps with draining material

d. Final shape of the embankment

Figure 1. The construction steps of the embankment

The site lithology (Figure 2), as it has been revealed both by the geotechnical report used for design and by two additional reports following the occurrence of the instability, shown an inclined layering, quasi-parallel to the original slope (less than 8º angle), alternating cohesive (layer 1 and 3) and cohesionless (layer 2) soils. The infill material was a compacted mixture mostly made of the soil from layer 1. Of an outmost importance is the fact that the cohesionless layer 2 has a larger thickness uphill than downhill. Combined with the heavy rain-flow regime of the area (during the landslide, more than 150mm / 24h) and the fact that the natural slope actually collected a large catchment surface, led to the fact that the intake discharge was almost double with respect to the outlet, quickly saturating the layer and increasing the pore water pressure to a highly artesian level.

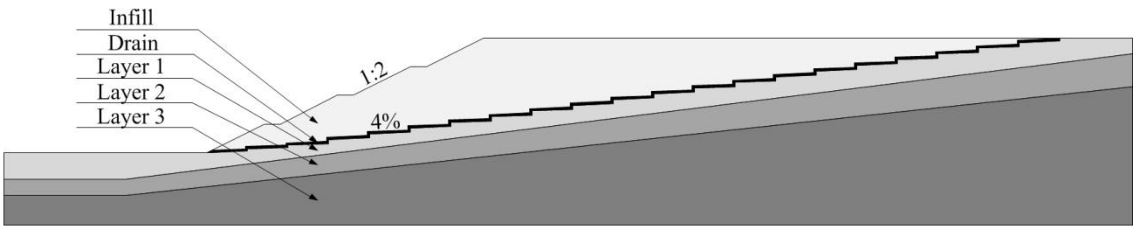
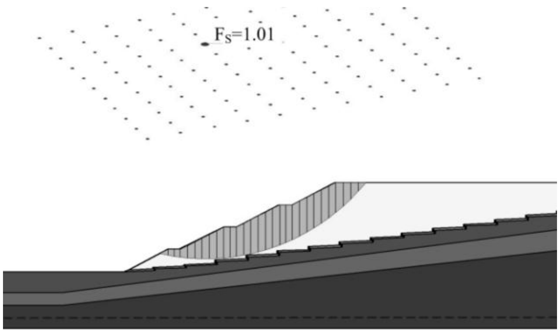


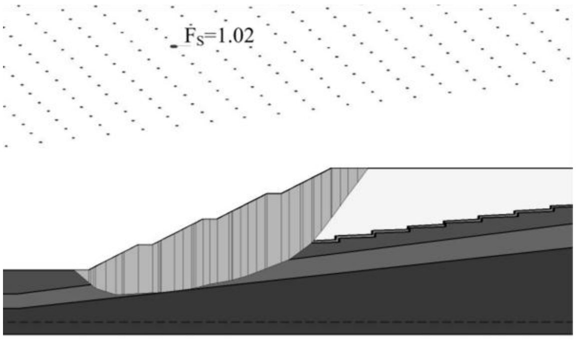
Figure 2. Cross-section of the embankment

The original design carried out in 2006 was based on the factor of safety approach, pre-dating the implementation of Eurocode 7 provisions. The site was completed in 2008 and it was in full service until 2010, when the landslide occurred.

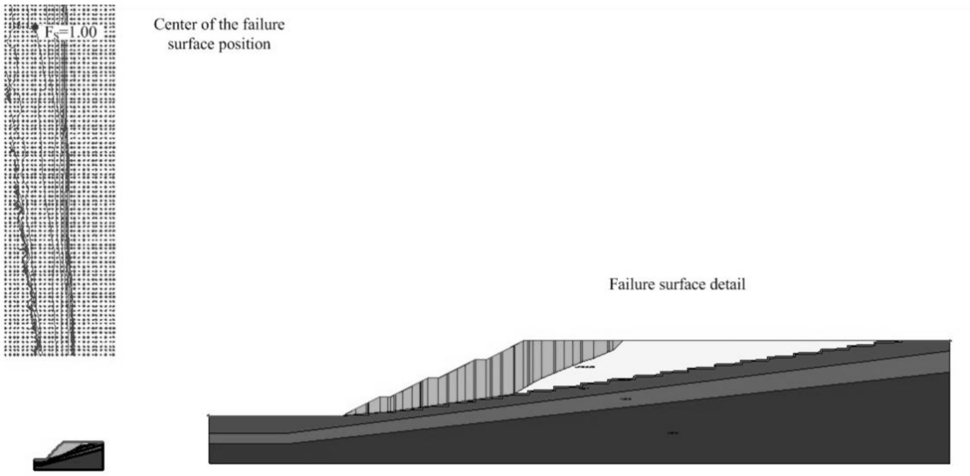
An initial step in the assessment of the instability causes was to identify the failure mechanism and the layer actually triggering the phenomenon. The back-analysis started from the characteristic values reported by the designer, decreasing artificially the shearing strength parameters of each layer in order to match the failure shape (Figure 3) with the one recorded on site (Figure 4).



a. Failure through the infill (unconfirmed)



b. Failure through the boundary between the layers 1 and 2 (confirmed in-situ)



c. Failure through the drain (unconfirmed)

Figure 3. The possible failure patterns

Figure 4. Picture of the actual landslide

The most plausible failure surface was identified to be passing through the cohesive layer 1, at the boundary with the water-bearing layer 2, most probably due to a combined effect of the additional horizontal thrust induced hydrostatically by water raise and the decrease in the effective shearing strength.

NUMERICAL MODELLING AND INPUT PARAMETERS ANALYSIS

Taking into account that the numerical modelling was based on a forensic analysis, two approaches have been employed. Noticing that the additional geotechnical reports fit within the same range of values as the initial design report, it was decided to combine all the available information and to process them in order to reduce the effect of the values bearing excessive errors.

All computations have been carried out without applying safety coefficients, since the aim of the analyses was to match the actual scenario, and they were not meant for design. The first approach was to consider the mean values resulted from the geotechnical investigations, while the second was a stochastic analysis using normal distribution for the soil parameters, as recommended in literature [16].

Obtaining the shearing strength parameters for the infill was performed, in the case of the deterministic calculus, by averaging, in a balanced way, the parameters obtained from triaxial tests carried out during construction by means of the least squares method (Figure 5), with the final shearing strength parameters values from the subsequent geotechnical reports, for which the determination method was not reported.

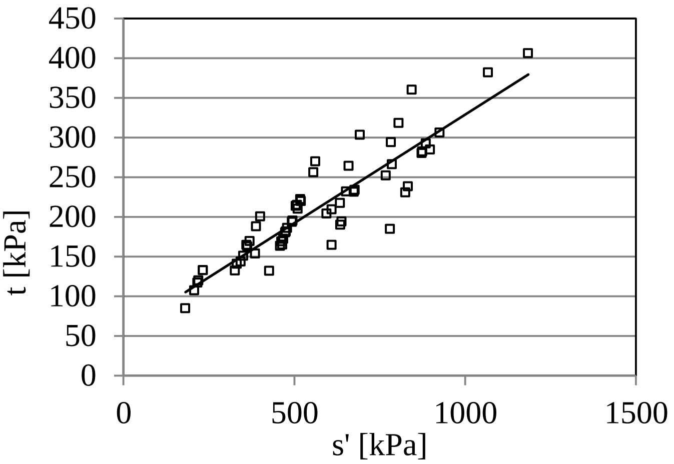


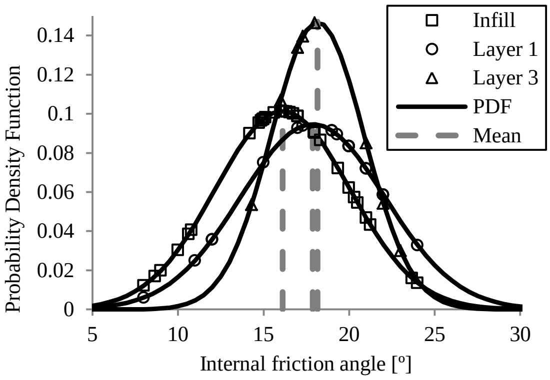
Figure 5. Least squares method employed on the triaxial tests results

The deterministic average approach supplied computation parameters for all the layers, even for those where only scarce information was available (such as the case for the cohesionless layer 2). For this particular layer, where the sampling and the subsequent laboratory testing were impossible, the linear deformation modulus was derived from the SPT tests carried on in-situ, while the permeability coefficient was approximated by empirical equations as a function of the material grain size distribution. The merging steps drain was considered to have the same parameters as layer 2, since it was laid from the same material, taken from a borrow pit on the site. The average values for all significant characteristics are reported in Table 1. Poisson’s ratio is not reported in any documentation and it was approximated from the literature, starting from the nature and the state of each soil type.

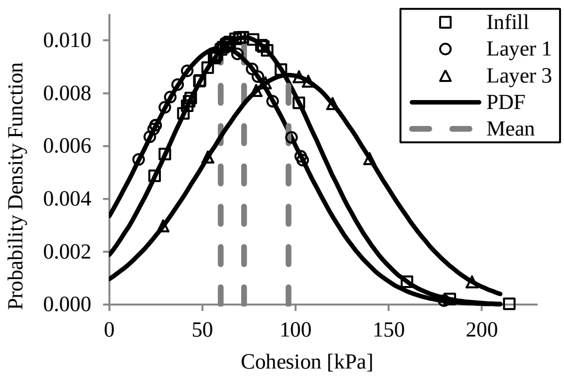
Table 1. Soil parameters used for stochastic analyses

Soil type	**γ [kN/m³]**	**ϕ [º]**	**c [kPa]**	**E [kPa]**	**ν [-]**	**k [m/s]**
Infill	19.5	16	70	16954	0.32	1.9E-11
Layer 1	20.8	18	60	18139	0.35	1.69E-11
Layer 2	20.0	23	7	20000	0.30	1.00E-05
Layer 3	20.1	18	96	22342	0.38	1.66E-11

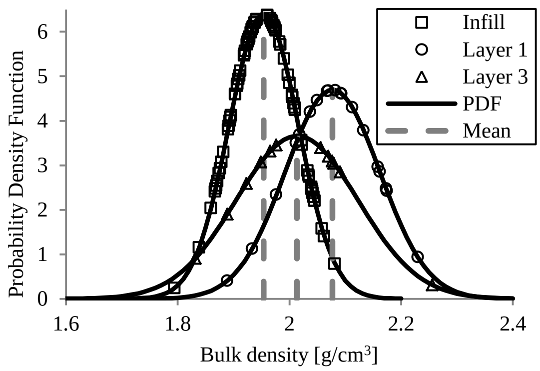
The probabilistic approach was carried out for all the parameters and the layers where the information was sufficient and necessary for limit equilibrium method, namely the shearing strength parameters (Figure 6 a and b) and unit weight (Figure 6 c) for the infill and layers 1 and 3. The computation has been carried out assuming a normal distribution for every parameter, as suggested in literature [16], deriving the probability density function (PDF) leading to the mean values and standard deviations (Figure 6 d).



a. Probability density functions for the internal friction angle



b. Probability density functions for the cohesion



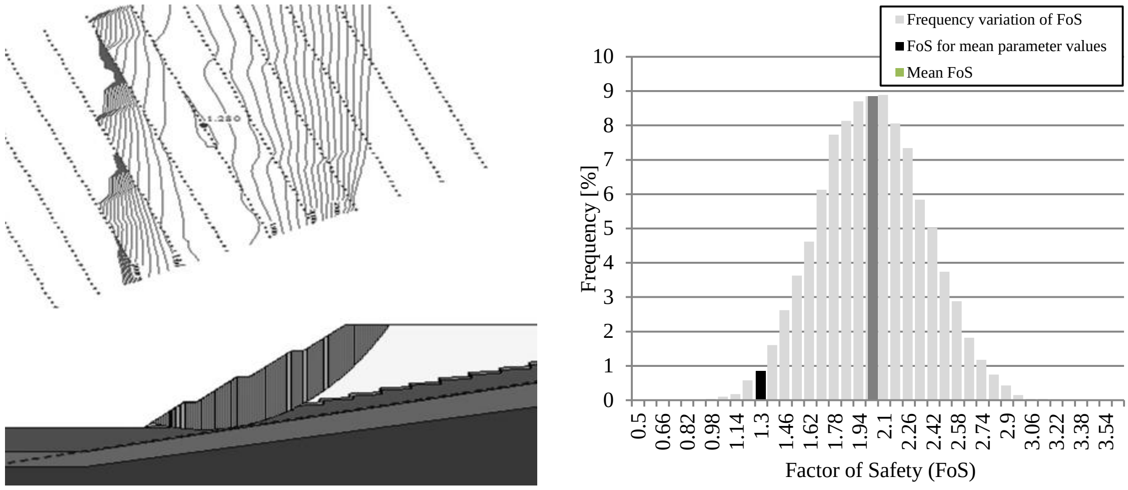
c. Probability density functions for the bulk density

Soil type	γ [kN/m³]		ϕ [º]		c [kPa]	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
Infill	19.54	0.62	16.10	3.93	72.35	39.46
Layer 1	20.77	0.85	17.87	4.21	59.89	41.05
Layer 3	20.14	1.09	18.16	2.72	96.20	45.92

d. Parameters used for stochastic computation

Figure 6. Statistical analysis of the soil parameters

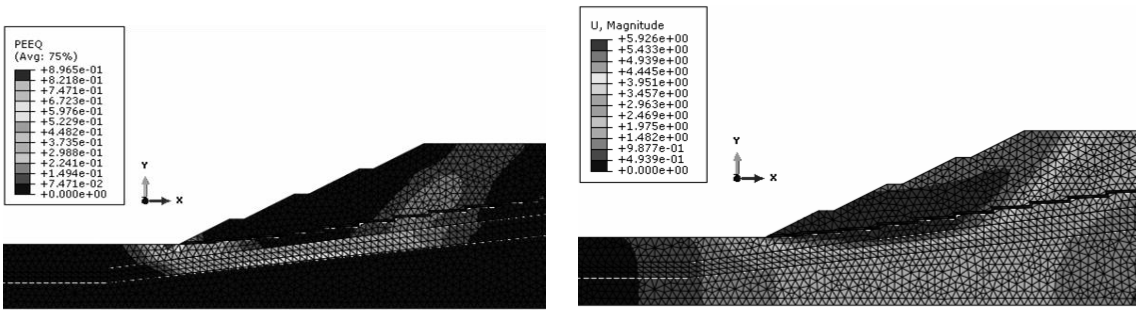
The limit equilibrium analysis performed by Janbu method (Figure 7) led to a critical safety factor of 1.28, while the mean safety factor was computed to be 2.07. The obtained probability of failure was 0.02%. Due to the variation of parameters, which were within a proper level of confidence, the stochastic analysis indicates a very stable soil mass, even more optimistic than the deterministic method. The reality showed, however, that the embankment failed and no subsequent geotechnical reports could prove the mechanical parameters to have been chosen non-conservatively. In order to find the proper failure mechanism, the next analysis step was to employ a full finite element method computation, including staged consolidation and sudden increase of the groundwater table.



a. Critical factor of safety for mean values b. Factor of safety frequency distribution

Figure 7. Results of stochastic analysis with limit equilibrium method

The triggering of the landslide, perfectly corresponding to the actual phenomenon was obtained even after two years consolidation stage by suddenly increasing, in transient step, the pore water pressure in the layer 2. The development of plastic zones initiated the formation of the failure surface in a progressive manner, propagating the failure with the evolution of displacements, which ended with values equivalent to collapse (6m).



a. Development of plastic zones b. Total displacements

Figure 8. Results of finite element method using deterministic approach

CONCLUSIONS

By the case history described herein, it was shown that in some cases, simplified numerical methods cannot take into account some unique site conditions that may occur with a very low probability. Even a stochastic approach over the mechanical parameters cannot compensate the stress evolution in a composite soil mass and the loads variation in a transient analysis. One has to bare in mind that all the computations showed in the paper were performed post-factum, knowing that the embankment failed as well as the shearing surface. The conclusion is that the employed simplification assumptions in the analyses must either be conservative, or have a very high confidence level, if non-conservative. The provided real-life example has shown a very specific case when the stochastic analysis may be non-conservative with respect to deterministic one, which, at its turn, is non-conservative with respect to a fully coupled transient finite element analysis, and, of course, the in-situ facts.

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